Bee-line 2

Question

A bee is sitting at a distance, d, from a man. At time t=0 the bee pursues the man at a constant acceleration of 1. The man runs in a straight line away from the bee with an acceleration of 0.3t. What is the largest initial separation, d, for which the man can't quite escape the bee?

Solution

Focus on the *relative* accelerations, velocities and distances between the bee and the man. The acceleration of the bee relative to the man is given by:

$$a_{rel} = 1 - 0.3t \tag{1}$$

With both bee and man at rest to begin with, we integrate equation (1) with respect to time to get the *velocity* of the bee relative to the man at time, t, as:

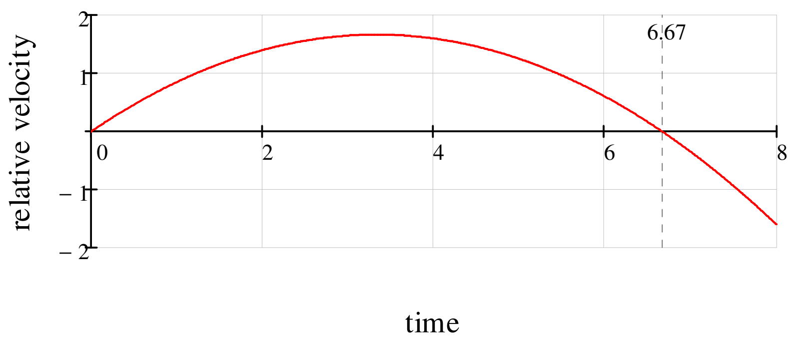
$$v_{rel} = \int_0^t a_{rel}dt = \int_0^t \left(1 - 0.3t\right)dt = t - 0.15t^2 \tag{2}$$

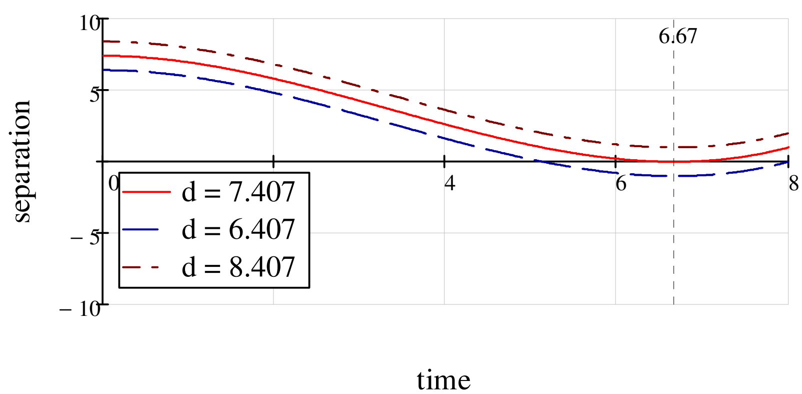
Integrating again, we find the distance of the bee relative to the man at time t to be:

$$x_{rel} = d - \int_0^t \left(\tau - 0.15\tau^2\right)d\tau = d - 0.5t^2 + 0.05t^3 \tag{3}$$

Clearly, at the point where the bee catches the man, the relative separation is zero. Also, if d is sufficiently large that the bee only just catches the man (i.e. any larger and the man would escape, any smaller and the bee would catch the man sooner) then the relative velocity is also zero at that point (to convince yourself of this, study the graphs of relative velocities and separations below).

From equation (2) we find that the relative velocity is zero when $t = 20 / 3 \approx 6.67$, and putting this value into equation (3) and setting the final separation to zero we find that the maximum possible initial separation is $d = 200 / 27 \approx 7.407$.


relative velocity
time
6.67
0
2
4
6
8
2
1
− 1
− 2


separation
time
6.67
10
5
− 5
− 10
0
4
6
8
d = 7.407
d = 6.407
d = 8.407