

ASSESSMENT OF COMPLEX, NON-STRUCTURED MATHEMATICAL PROBLEMS

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Abstract: Learning outcomes are considered to be a key tool for student-centered teaching and learning. They can be successfully implemented in teaching and learning mathematics on higher educational level and together with appropriate level of technology enhanced learning can provide the framework for successful learning process even for students that have not been primarily interested in mathematics. One of the most prominent goals of mathematics in non-mathematics major study programs is providing students with mathematical modelling skills. In this respect assessment methods should be carefully chosen and implemented in concrete learning context to take into account problem posing and problem solving. This paper describes the way in which rubrics can be used in a mathematical course in order to assure the fulfillment of the learning outcome concerning mathematical modeling and problem solving. Problem posing out of the real world context confirmed to be hard task for students.

Keywords: mathematical modelling, e-assessment, rubrics, learning outcomes, problem solving

1. Introduction

Lifelong learning and competences linked to employability have become a necessity in today's society characterized by rapid social, technological and economic change. Problem solving skill together with teamwork are highly appreciated by today employers (Singh et al, 2014, Divjak & Spahic, 2008) and belong to the key competences emphasised in all qualification frameworks, particularly European Qualification Framework - EQF (EQF, 2008) and researches on future development of job market (Frey & Osborn, 2013). Further, (Sodhi, M., Son B-G., 2007) analysed 401 mathematical modelling-related job advertisements to find out what employers want from graduates from mathematical modelling related courses and showed that many skills required for mathematical modelling jobs are "soft" skills relating to problem solving and to communication, which are not covered by conventional mathematical modelling curricula. Therefore, they recommend that "Educators can help students obtain such soft skills by adopting innovative teaching methods such as team exercises, competitive exercises, group discussions, presentations, and case analysis." Opposite to that, in academia environment very often the implicit assumption is that those skills of analysing, structuring and modelling real life situation would develop as a side effect of education and learning tools and techniques. Our hypothesis is that it is not the case. It is certainly not the only goal of mathematical education. Surely, students should acquire mathematical way of thinking and know how to present mathematical argument both orally and in writing. At the same time, students should spent more time on mathematical modelling and problem solving and using current technology doing that (Holton, 2005).

Using technology and especially e-learning has been considered crucial for support of formal education and enabling of lifelong learning in higher education (Garrison, 2011). Unfortunately, majority of technology enhanced learning (e-learning) does not cope well with deeper learning (Entwistle, 2009, p 138) and try to use new technology without rethinking educational goals and pedagogical practices (Garrison, 2011). Therefore, a case study of good practices of enhancing of

problem solving skills in e-learning environment as well as research on drivers enabling students to develop creativity and innovation is well justified (Divjak, 2012). Assessment guides learning and therefore it should be considered and constructed carefully but at present, there is a lack of a pedagogical vision of how to develop 21st century skills with an adequate use of ICT and ICT-based assessment (Redecker & Johannessen, 2013). Assessment procedures in formal higher education have traditionally focused on examining knowledge and facts through formal testing (Cachia et al., 2010) and very rarely ask for and support problem solving in authentic environment. Finally, let us stress that enhancing students' problem solving skills is not in competition with their acquiring mathematical theories and abstract thinking. On the contrary, these two can successfully complement each other as this paper hopefully confirms.

2. Non-structured mathematical problems and assessment

Learning outcomes are considered to be a key tool for student-centered teaching and learning. They can be successfully implemented in teaching and learning mathematics within higher education and together with appropriate level of technology enhanced learning they can provide the framework for successful learning process even for students that have not been primarily interested in mathematics (Divjak, 2012). In order to illustrate and analyse that approach one case study will be presented. The course Discrete Mathematics with Graph Theory (DMGT) is taught in the first year of master level of study programs Information Systems and Software Engineering at the Faculty of Organization and Informatics, University of Zagreb. It is taught as a blended learning course because it is conducted both online and on campus. The syllabus consists of two parts: in the first part different topics in discrete mathematics are covered and the second half is dedicated to the graph theory and its applications. The topics have sound foundations in mathematical theory but offer multitude of applications of the covered theory in computer science and business, e.g. problem solving exercises that are performed individually or in teams.

Probably the most prominent goal of teaching mathematics in non-mathematics major study programs is providing students with mathematical modelling skills (Abramovich and Grinshpa, 2008) having in mind that "mathematics is both a body of knowledge and a creative process" (Holton, 2005). Development of modelling skills is the task that needs to be implemented throughout the course design and implementation, but mostly by using interesting and well prepared assessment methods. Assessment guides student learning, but assessment methods are rarely the focus of interest among educators and curriculum designers.

On the other hand, a deep learning is enhanced when assessment procedures emphasise and reward student understanding (Entwistle, 2000) and when assessment methods are clearly connected with intended learning outcomes of the study program or the course (Biggs, 2003.). The constructive alignment for the course DMGT is presented in Table 1. The term constructive alignment is coined by John Biggs in short it means learning outcomes must be aligned with teaching, learning and assessment as well.

<i>Study program learning outcome - relevant for the course</i>	<i>Course specific learning outcome related to the study programme learning outcome</i>	<i>Teaching and learning method</i>	<i>Assessment method</i>	<i>Student workload - ECTS credits</i>
Apply mathematical methods, models and techniques appropriate for solving problems in the field of information and business systems	Solve real world problems in ICT with methods from graph theory and discrete maths (individually and in teams)	Students work in teams of three on posing and solving authentic problems – online work	Teacher assessment and partially peer assessment of two stages of problem posing and problem solving based on prepared criteria and scoring rubrics	40 hours = 1.5 ECTS (approx. 20% of the course 7 ECTS)

Table 1. Constructive alignment on the course DMGT

Problem solving is usually regarded as the thinking and behaviour we engage in to obtain the desired outcome we seek (Treffinger et al., 2008). The outcome could be attaining a certain goal or finding a satisfactory answer to our question. Besides classical problem solving, when the description of a problem is given to students by teachers, we try to develop additional student's competence connected with recognizing real life non-structured problems that can be formulated and afterwards solved by the use of non-trivial mathematical theories and techniques which students have learned in the course. In such a case students become problem owners replacing the industrial or public representatives in so-called client-driven projects (Maki et al., 2006) and they are interested to formulate it carefully and also to monitor the solution finding process, as well as to evaluate the final solution. This teaching method actively engages students in a deep conceptual mathematical activity and develops their mathematical reasoning and collaborative learning. Collaborative learning is all about sharing knowledge but there must be common ground, language, joint focus and compatible perspective (Stahl & Hesse, 2009). It is very important to explain the educational goals of the activity at the beginning of collaborative work and to provide students with the joint (online) space for sharing and developing ideas. The wiki and the virtual reality were designed to create shared perceptual space, where salient objects could be seen by all.

Therefore the learning process begins with the first phase where students analyse real life situations in teams and try to recognize potential problem that can be modelled and later solved by another team by using the theoretical frameworks they were taught during the course. In this first phase called Problem posing students have very complex task to identify the authentic problem having in mind theoretical mathematical frameworks and practical issues. The first phase usually lasts a month. After that teachers need a week to read and analyse all the proposed problems, give feedback and ask further clarifications. Then teachers redistribute the problems between teams. In the second phase called Problem solving another team should solve a posed problem. Finally, all teams have to submit their solutions and present them in written (in wiki in LMS Moodle) and oral form (if they obtain acceptable solution). Final grading is done by teachers but students from the team that posed the problem are invited to comment on the presented solution and in that way can influence final grade. The whole process is presents in Figure 2.

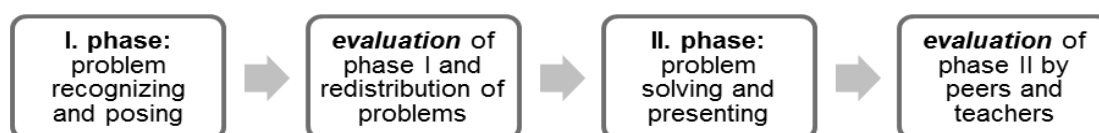


Figure 2. Phases in problem solving on DMGT

3. Assessment of non-structured problems

It is widely accepted that assessment should be based on well-defined assessment criteria but it is very rarely explained how to define, implement and evaluate those criteria. Rubrics can be successfully used as a tool in that process. In the process of building comprehensive and consistent rubrics there are two essential steps that have to be carefully considered. Firstly, it is a process of formulating and describing assessment criteria that are clearly connected to defined learning outcomes. Secondly, it is a process of determining weights of assessment criteria that take into account different perspectives (teachers', students', employers', other experts' & stakeholders'). Once assessment criteria and learning outcomes are defined, they can also be easily implemented as a rubrics in a virtual environment and used in e-learning. Well defined rubrics communicate to students what is asked from them and support their reflection and critical skills (deep learning).

Rubrics are especially useful when more than one teacher is involved in the process of assessment and grading and when combination of teacher and automated grading is implemented. Finally, rubrics are vital in a case of a complex task including problem-based learning, group work or peer assessment that are authentic to the skills being tested.

This paper recommends a comprehensive method to prepare, moderate and execute two above mentioned steps that, beside sound pedagogical approach, incorporate the use of multicriteria decision making method. The Analytic Hierarchy Process (AHP), introduced by (Saaty, 1980), is a multicriteria decision making technique that supports group decision making performed by heterogeneous group. There are successful implementations of multicriteria decision making in the context of higher education, especially the AHP in e-learning (Begicevic, Divjak and Hunjak, 2011; Begicevic, Divjak and Hunjak, 2007).

Let us describe the way in which we used rubrics in DMGT course in order to assure the fulfillment of the learning outcome concerning mathematical modeling and problem solving. In the Table 1 the constructive alignment is presented and it can be seen how learning outcomes and teaching and learning as well as assessment method is connected to the student workload. Namely, real problem posing and solving contributes amounts to 20% of the overall course grade (student workload of 30-40 hours). Since the total grading on the course is 100 points it means that we attached 20 points to the problem solving task. From the outset it is important to recognize the assessment criteria and then to attach grades and levels describing certain grading to each criteria. In our case of problem posing and solving we first have to weight relative significance of these two phases having in mind the intended learning outcomes. It was decided to introduce the ratio 2:3 between phases I and II i.e. 8 points to criteria for the problem posing phase and 12 to the problem solving phase. We justified that by group decision making using AHP and the decision makers were: one professor, two research assistants and one former student. Their judgements were equally weighed during the decision making, exercise was performed anonymously and afterwards the judgements were aggregated. In the next step it was essential to recognize the criteria that would appear in the scoring rubrics. In the first phase we identified three main criteria:

Criteria 1: Problem relevance

Criteria 2: Problem description

Criteria 3: Characteristics of solution.

In the second phase six criteria were distinguished:

Criteria 4. Linking given problems with theory of DMGT (modelling),

Criteria 5: Strategy of solving

Criteria 6: Characteristics of the solution

Criteria 7: Implementation of the solution and interpretation

Criteria 8: Written and oral presentation

Criteria 9: Teamwork.

After that the relative importance of different criteria can be determined again by group decision making using AHP. The final result for the scoring rubrics together with description of the levels is presented in the Appendix. It can be noticed that at the end the weights of the criteria were slightly modified. Finally, description of the levels is necessary in order to enable consistent grading especially when there is more than one teacher included in the course.

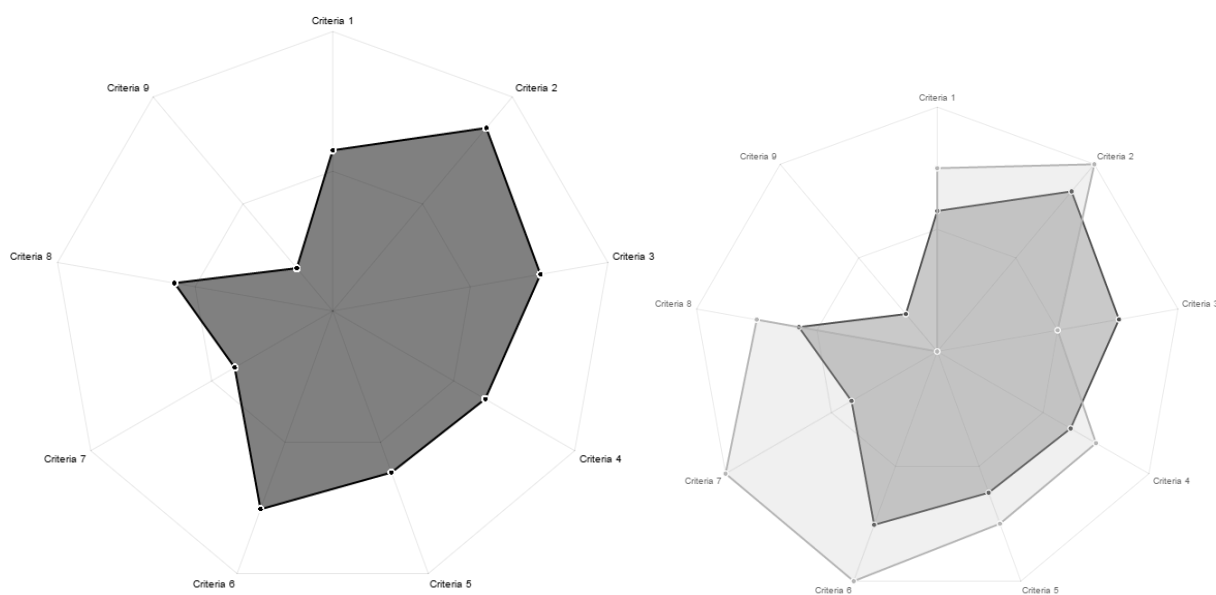
However, this process can be extremely demanding for a teacher to follow and assess especially with a big group of students. Luckily, there are ICT tools and social software available that can enhance and enable implementation of such a challenging problem solving exercise. Therefore students need virtual learning environment that allows them to discuss potential real world problems suitable to be described, modelled and solved by some mathematical theory or approach, construct mathematical solutions collaboratively and exchange references in situations where they are not co-located. More details on the implementation in Moodle and use of social software can be found in paper (Divjak, 2012).

Students' works are assessed and graded by rubrics implemented in Moodle and therefore there are the extended set of data on students achievements due to the criteria defined in advance. Importantly, there is a possibility to find out if the students succeeded in capturing the idea of modelling real life situation and in which stages students need more support by teachers and/or by their peers.

4. Results and discussion

Finally, it is worthwhile to evaluate the level of general achievement of a particular learning outcome that appears at the study program level as well as on the course level. It can be done more analytically if we use the criteria in the rubrics Results for the DMGT course of 2014/2015 will be briefly presented in this section. It is important to emphasise that problem posing and solving task is and discussed used in the course as formative and summative assessment because it is used to shape and improve students' competences, because mathematical modelling in itself is knowledge construction, but at the same time it is used for summarising students' achievements.

In the academic year 2014/2015, 104 students were enrolled in the DMGT course and approximately one fourth of them were re-enrolled because he/she did not successfully complete the course the previous year. There are two ways for successful completion of the course: the first that students collect 50% of points during the semester and the second to successfully complete written and oral exams during summer, autumn or winter examination periods. Students averaged 44.84 points out of 100 during continuous formative grading and 50% of them passed the course due to their work during semester. The problem posing and solving task had an alternative in the form of taking appropriate MOOC (Massive Open Online Course) and 9 students took that alternative. Additionally, students that re-enroll the course can transfer last year's scoring on problem solving if the score is acceptable. Finally, 81 students divided into 28 teams work on the problem solving assignment and the average result was 10.75 out of 20. In the Graph 1 the overall results are visualized according to the criteria from the rubric. Additionally, each student has a possibility to see his/her own results and compare it with the average (example in the Graph 2). We prepared and implemented that tool into LMS Moodle in the academic year 2014/2015 in order to give students the possibility to better monitor their progress as well as to see where they are above average and in which areas they might invest more time and effort. In this way the e-assessment tool is enriched to cover all stages of the problem posing and problem solving: online collaboration (wiki and social software for bookmarking), e-assessment (online rubrics) and reflection (visualization of results in form of radar chart).



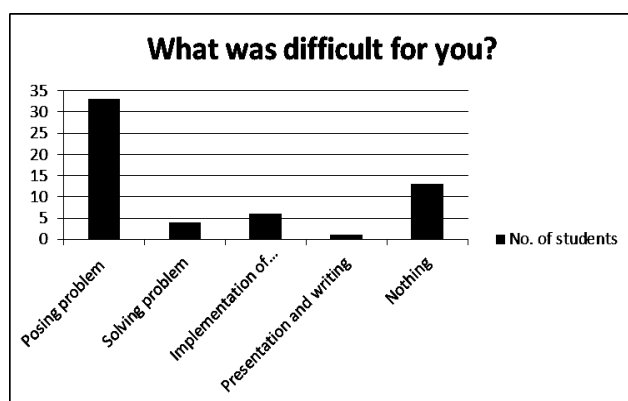
Graph 1. (left) Radar chart of average students' grades per criteria on the course DMGT 2014/2015

Graph 2. (right) A student scoring comparison with the average scoring of the group

In order to understand the results presented in Graph 1 it is important to note that only those teams that scored more than 5 points in first four criteria of Problem solving phase are invited to present their solutions orally and get extra bonus on Teamwork criteria. This complicates the analysis of the last two criteria. From the Graph 1 it is obvious that in the first phase the criterion with the lowest average grade is Problem relevance. In the second phase Problem solving, if we set aside last two criteria that

were specifically scored, the lowest average result was for Problem implementation and interpretation. These findings give us very important information about student's ability to work in the real life situation and to apply what they have learnt in theory, especially within mathematical theories.

Finally, students were invited to fill out an anonymous online questionnaire and evaluate teaching and learning process on the course as well as their own achievement in a form of answering close type questions but also in free form. In total, 58 students out of 104 used that opportunity. Let us briefly present their answers to the questions relevant for the topic of problem posing and solving. One of the questions asked students what had been difficult for them in the problem posing/solving project work and result is shown in Graph 3.



Graph 3. Students' answers to the question: What was difficult for you in the task of problem solving?

There are also many comments in free form and there are two typical comments: *"Idea of project work is ingenious and it is obvious which teams really work hard on it, show mathematical knowledge but also can solve real problems. This kind of approach is missing generally in mathematics"* and *"I think it should be more clearly defined what is being asked in this task, put 2-3 good examples on the Moodle to give us a clue what is expected."* Students' answers and comments clearly show that recognizing a problem in the real context and connect it with the theory is hard for them and in that way they confirmed what has already been detected by analyzing scores in the rubrics. It demonstrates also that students are used to mostly copying or adopting given problems and solutions without creativity and innovation. Moreover, the majority of them recognize the importance of learning problem posing and solving as well as of the mathematical modelling.

5. Conclusion

The importance of the way we teach and learn mathematics is increasing nowadays. Especially when we teach mathematics to non-mathematics major students who lack intrinsic motivation for learning mathematics. In that case two main goals should be fulfilled: development of students' abstract thinking, mathematical reasoning and enhancing students' modelling skills. In the later, a special attention should be given to development of authentic tasks for students that are aligned with an intended learning outcome of the study program and consequently of the course. Mathematical modelling begins with problem posing i.e. in recognizing the potential problem solving task in an authentic situation that can be modelled and probably solved using certain mathematical theories and tools. That phase needs to be further investigated and researched.

The case study presented in this paper shows that students are lacking skills in identifying relevant problems that can be connected with mathematical theory and skills to implement solutions in form of algorithms, software as well as to interpret results. It is important to further research those aspects. Furthermore, the course is delivered in a blended learning mode and learning and teaching environment is constructed in a way to support e-assessment of complex non-structured problem solving accompanied with the detailed scoring rubrics and visualisation of group results as well as benchmarking of students' results. In that way, the assessment is transparent, learning-oriented and consistent. Finally, students are aware of the demand for solving non-structured real-world problems and even if they are not mathematics majors they appreciate mathematical modelling as a useful skill. Therefore, important task of mathematics teachers is to explain the role of mathematics in certain

study programmes in developing abstract and logical thinking as well as to help students to enhance their problem solving skills and through it deepen and strengthen their understanding of mathematics.

References

- Abramovich, S. and A. Grinshpa (2008). Teaching Mathematics to Non-Mathematics Majors through Applications, *PRIMUS Problems, Resources, and Issues in Mathematics Undergraduate Studies*, vol 18 (5) pp. 411-428.
- Begicevic N., B. Divjak and T. Hunjak (2011). AHP-based group decision making using keypads, *International Journal of Economics and Business Research*, vol 3 (4) pp. 443-458.
- Begicevic N., B. Divjak and T. Hunjak (2007). Prioritization of e-learning form: a multicriteria methodology, *Central European Journal of Operations Research*, vol 15 (4) pp. 405-419.
- Biggs, J. (2003). *Teaching for Quality Learning at University – What the Student Does 2nd Edition* SRHE / Open University Press, Buckingham.
- Cachia, R., A. Ferrari, K. Ala-Mutka and Y. Punie (2010). *Creative Learning and Innovative Teaching: Final Report on the Study on Creativity and Innovation in Education in EU Member States*, Seville, JRC-IPTS.
- Divjak, B. (2012). Chapter: Implementation of learning outcomes in mathematics for non-mathematics major by using e-learning in the book: *Teaching Mathematics Online: Emergent Technologies and Methodologies*, Editor(s): A. A. Juan M. A. Huertas S. Trenholm C., Steegmann; IGI Global.
- Divjak B. and A. Spahić (2008). Linking Higher Education and Employers: Bologna Reforms and the Croatian Private Sector, *Higher Education in Diverse Communities, Global Perspectives – Local Initiatives*, Ed. F. Ferrier and M. Heagney. Higher Education Authority London pp. 105-110.
- EURYDICE (2012). *Developing Key Competences at School in Europe: challenges and opportunities for policy* (Education, Audiovisual and Culture Executive Agency).
- European Qualification Framework for Lifelong Learning. European Communities (2008). Available at: http://www.ond.vlaanderen.be/hogeronderwijs/bologna/news/EQF_EN.pdf
- Entwistle, N. (2000). Promoting deep learning through teaching and assessment: conceptual frameworks and educational contexts. Paper presented at the TLRP Conference, Leicester, November 2000.
- Entwistle, N. (2009). *Teaching for Understanding at University. Deep Approaches and Distinctive Ways of Thinking*, Palgrave MacMillan.
- Frey C. B. and M. A. Osborne (2013). *The Future of Employment: How Susceptible are Jobs to Computerisation?*, Available at: http://www.oxfordmartin.ox.ac.uk/downloads/academic/The_Future_of_Employment.pdf
- Garrison, D. R. (2011). *E-learning in the 21st Century: A Framework for Research and Practice*. Routledge Taylor & Frances Group. NY.
- Holton, D. (2005). Tertiary mathematics education for 2024. *International Journal of Mathematical Education in Science and Technology*, vol 36 (2-3) pp. 305-3016.
- Maki D., W. Winston, M. Shafii-Mousavi, P. Kochanowski, C. Lang, K. Ernstberger and T. Hodgson (2006). On the use of client-driven projects in the mathematical classroom, *PRIMUS Problems, Resources, and Issues in Mathematics Undergraduate Studies*, vol 16 (4) pp. 300-3013.
- PISA (2014). *PISA 2015 Draft Mathematical Framework*, available at OECD Published on 2014-03-18 File name: draft%20pisa%202015%20mathematics%20framework%20.pdf
- Redecker, C. and O. Johannessen (2013). Changing Assessment - Towards a New Assessment Paradigm Using ICT, *European Journal of Education*, 48 (1), pp. 79-96.
- Saaty, T.L. (1980). *The Analytic Hierarchy Process, Planning, Priority Setting, Resource Allocation*, McGraw-Hill, New York.
- Sodhi, M. and B.G. Son B. (2007). Math Modelling: What Skills do Employers Want in Industry? *Mathematical Modelling*, pp. 212-220.
- Stahl, G. and F. Hesse (2009). Paradigms of shared knowledge, *International Journal of Computer-Supported Collaborative Learning*, 4(4), 365-369.
- Treffinger, D. J., E. C. Selby and S. G. Isaksen (2008). Understanding individual problem-solving style: A key to learning and applying creative problem solving, *Learning and Individual Differences*, 18, 390-401.

Appendix. Scoring rubrics for DMGT

Criterion	Level of achievement				
	Poor (0)	Acceptable	Good	Excellent	P
Phase I – Problem posing					
Problem relevance	Problem is copied or not relevant.	Problem is standard for DMGT. (1)	Problem is interesting and connected with DMGT. (1.5)	Problem is challenging & interesting and can be connected to DMGT. (2)	2
Problem description	Problem is not clearly described or it is not a problem.	Problem is described but without additional data, links or real context. (1)	Problem is described clearly but there is no important (relevant) real context. (2)	Problem is described in clear and interesting way with relevant real context. (3)	3
Characteristics of solutions	It is not clear what is asked from team that is going to solve it.	There are elements of characteristics of solution, but the framework for acceptance for solution is not given. (1)	Characteristics of acceptance of solution are given but there are no open questions about consistency and possibility of gaining solution of real world problem. (2)	Characteristics of acceptance of solution are given and there are still open questions about consistency, generalization and possibility of gaining solution of real world problem. (3)	3
Phase II – Problem solving					
Linking given problems with theory of DMGT (modeling)	There is no link between theory and problem.	There are hints about theoretical framework but correspondence and elements of modeling are not clearly given. (1)	There is well described theoretical model and modeling process but limitations and possible alternatives are not given. (1.5)	There is well described theoretical model and modeling process and limitations and possible alternatives are given.(2)	2
Strategy of solving	There is no given strategy of solving.	There are standard steps in solving the given problem but they are not verified. (1)	There are standard steps in solving the given problem and they are verified by using theoretical or experimental methods. (1.5)	Strategy of problem solving is innovative and clearly described and it is based on theoretical or experimental methods.(2)	2
Characteristics of the solution	Problem is not solved or given solution doesn't meet requirements.	Given solution partly meets requirements. (1)	Given solution meets all requirements but there is no analysis of limitations or no implementation of solution in real context. (2)	Given solution is innovative, meets all requirements and there is analysis of limitations and implementation in real context. (3)	3
Implementation of solution (software, tool, algorithm) and interpretation	There is no software or other implementation or it doesn't meet requirements.	There is implementation that partly meets requirements. (1)	There is standard implementation that fully meets requirements and problem characteristics. (1.5)	There is innovative implementation that fully meets requirements in a real world context. (2)	2
Written and oral presentation	Oral or written presentation was not prepared or they have not required elements.	There is written presentation but not according to requirements (references, links, structure) or oral presentation has not relevant or in the timeframe. (1)	Written presentation meets requirements. Oral presentation has all relevant elements and it is in the timeframe. (1.5)	Written and oral presentations meet all requirements and they are prepared in original and creative way to engage participants/readers. Social software is also well used. (2)	2
Teamwork	Not all team members have been involved in work.		All team members have been involved in work.(1)		1