

DEVELOPING AN INNOVATIVE CURRICULUM AND TEACHER PEDAGOGY FOR NURTURING A POSITIVE DISPOSITION TOWARDS LEARNING AND USING MATHEMATICS IN POST 16 STUDENTS IN ENGLAND – ‘CRITICAL MATHS’.

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Abstract: Research indicates many school-age learners have a negative disposition towards learning and using mathematics. This often then perpetuates into adulthood. Mathematics in Education and Industry are undertaking a project, funded by the Department for Education, to create a ‘Critical Maths’ curriculum. Critical Maths is for study post-16 and is suitable for students intending to enter a wide range of professional careers including social sciences, politics, and others where quantitative reasoning is considered important. This paper outlines the development to date, considers specific examples and pedagogies symbolic of Critical Maths and information of how UK Examination Boards have chosen to incorporate Critical Maths into their national qualifications is also given.

Keywords: Critical maths, curriculum, post 16.

1. Introduction

Mathematics in Education and Industry (MEI) is an independent charity and UK curriculum development body committed to improving mathematics education. Through dedicated funding from the Department for Education which began in 2012, MEI has been tasked with developing a new mathematics curriculum, based on Sir Timothy Gowers’ weblog (Gowers, 2012), under a project entitled ‘Critical Maths’. The curriculum needed to be suitable for students intending to enter a range of professional careers including social sciences, politics, and others where quantitative reasoning is considered important. Many of the higher education courses serving these professions do not require, as part of their admission criteria, students to have a mathematics qualification beyond grade C at GCSE (the qualification taken by most 16 year olds in England).

This paper will consider background research in the area before discussing the development of the materials, which includes the designing and trialing. The key area of ‘assessment’ of the curriculum will then be discussed alongside the related consideration of how the curriculum would be taught. Finally, information on how UK Examination Boards have chosen to incorporate Critical Maths into their national qualifications will be given.

2. Background

The report “Is Mathematics An Outlier?” (Nuffield Foundation, 2010) examined the level of post 16 mathematics education taking place in a number of countries. They concluded that the UK is almost unique in not requiring learners to study mathematics beyond the age of 16. The Wolf Report (Wolf, 2011), which reviewed vocational education, also raised concerns regarding the level of mathematics post 16.

Professor Sir Timothy Gowers from the University of Cambridge wrote an influential weblog “How Should Mathematics Be Taught To Non-Mathematicians?” considering the challenge of producing a curriculum suitable for post 16 learners, a curriculum which would take learners beyond GCSE whilst remaining relevant and motivating.

Making mathematics appealing to students who otherwise would not have studied it post 16 is one of the many challenges which the Critical Maths project has faced. There are many studies (Brown et al, 2008, Ashby, 2009, Epstein et al, 2010, Chinn, 2012) that present strong evidence that many students, from as young as Primary school age and going on into adulthood, have negative perceptions of mathematics. This is a long standing issue and over a decade ago in the national report “Mathematics Counts” (Smith, 2004) it was said that mathematics is perceived by many students as difficult when compared to other subjects. In addition course content was described as being perceived as boring, irrelevant and insufficiently stimulating.

It’s been found (Mujtaba et al, 2013) that the advice which students receive from teachers, family and friends, regarding the value of continuing to study mathematics post 16 is also influential. This is particularly concerning in light of the previously highlighted research that the perception of mathematics by a significant proportion of the public is negative, as also found in “Public Images of Mathematics”, (Sam, 2002).

Many of the points made above are also evident in the paper “Post 16 Mathematics: A Strategy For Improving Provision And Participation” produced by the influential Advisory Committee for Mathematics Education (ACME, 2012).

Professor Gowers’ weblog considered the subject content of a suitable curriculum, he commented on two articles, “What Is Mathematics For?” (Underwood, 2010), and “Discrete Mathematics And Public Perceptions of Mathematics” (Malkevich, 1997). Both of these articles raise questions about what proportion of the compulsory mathematics curriculum in the United States is actually used by a majority of citizens in their everyday life and career. When deciding on the mathematical content of the Critical Maths draft curriculum similar questions were asked. We also considered the ideas expressed in “Twenty tips for interpreting scientific claims” (Sutherland et al, 2013) and similar articles.

It was decided that if the curriculum produced was to be successful in countering these negative perceptions then it was paramount to ensure that the learning experience should, in part, be aimed at demonstrating the usefulness of mathematics. Particularly to the everyday experiences of the learners studying it. To achieve this the teaching materials should clearly show the value of mathematical thinking, for example when interpreting information in the media, making personal choices and professional decisions. It was also clear from (Smith, 2004) that any resulting qualification based on this curriculum would need to be perceived as valuable by Higher Education (HE) and employers as well as students.

Using Professor Gowers’ blog as a starting point, and drawing on the expertise and experience of the members of a Critical Maths advisory group a draft curriculum was proposed. This was well received in consultations with HE, employers and learned societies. What follows is a discussion of numerous aspects of the curriculum, including insight into the designing, trialing and assessment thereof.

3. Trialing the Materials

3.1 Designing teaching resources

Professor Gowers suggested an approach which starts with a simple but engaging question. At first the learner may have an impulsive answer, a little like system 1 as described in the book “Thinking, Fast and Slow” (Kahneman, 2011). Through discussion, skillfully prompted by the teacher, the learners would be steered towards thinking about the question in more depth, using Kahneman’s system 2. At such a stage the need for some quantitative reasoning, or logical thinking would introduce preselected elements of mathematics. As the discussion progressed the learner’s understanding of the question and the role of mathematics in helping to answer the question would grow.

This approach shared similarities with the ideas in “Beyond Show and Tell”, (Takahashi, 2008) as well as with the work of Freudenthal (Gravenmeijer and Terwell, 2000). Based on these philosophies resources were created. A simple example of one such resource and the methods used in the teaching of it will now be considered in detail.

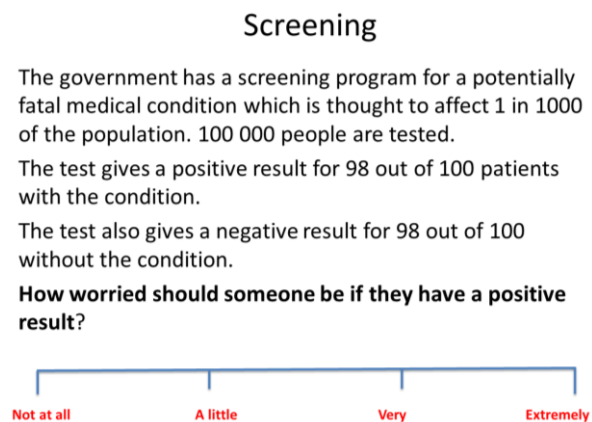


Figure 1 Example of a starter question.

The teaching starts by prompting an intuitive quick response, learners are asked to anticipate the level of anxiety they would expect someone to feel, if they received a positive result from a reasonably accurate medical screening test, for a potentially fatal condition. Learners are asked to illustrate this on a scale, then given the opportunity to confer with a partner or small group. In the classroom trials of this question the overwhelming first response is a point somewhere between very and extremely worried. When asked for a reason for this level of anxiety, most frequently the response refers to the accuracy of the test. The learners are then prompted to think a little more deeply with questions such as:

- Do you think there will be any people who tested positive but don't have the condition?
- Do you think there will be a lot of people in this situation?
- What are the different situations which people may find themselves in?
- Is it possible to estimate how many people we would expect to be in each of these situations?

Learners are asked to work on the final question listed in the bullets, whilst the teacher circulates and watches, looking at some of the different approaches the learners use. When the teacher feels the students have made sufficient progress with the task, the different approaches are compared and contrasted. A typical solution to the question is shown in figure 2.

Students are asked to review their original feelings regarding the level of anxiety with the additional perspective provided by the mathematical analysis of the situation. This can be prompted further by questions such as:

- Are there more true, or false positive results?
- What is the probability that someone who tests positive actually has the medical condition?

Students can be prompted to think more about the context, with questions like – The test makes two different types of error, a false positive and a false negative, which is the most worrying in your opinion, why? This could be extended, in each case, by asking, what can be done to limit the risk?

In the trials this has raised issues such as, undue worry, a patient receiving unnecessary treatments from false positives, and, in the case of false negatives, the patient not being treated in sufficient time to save their life.

Screening Info Spreadsheet

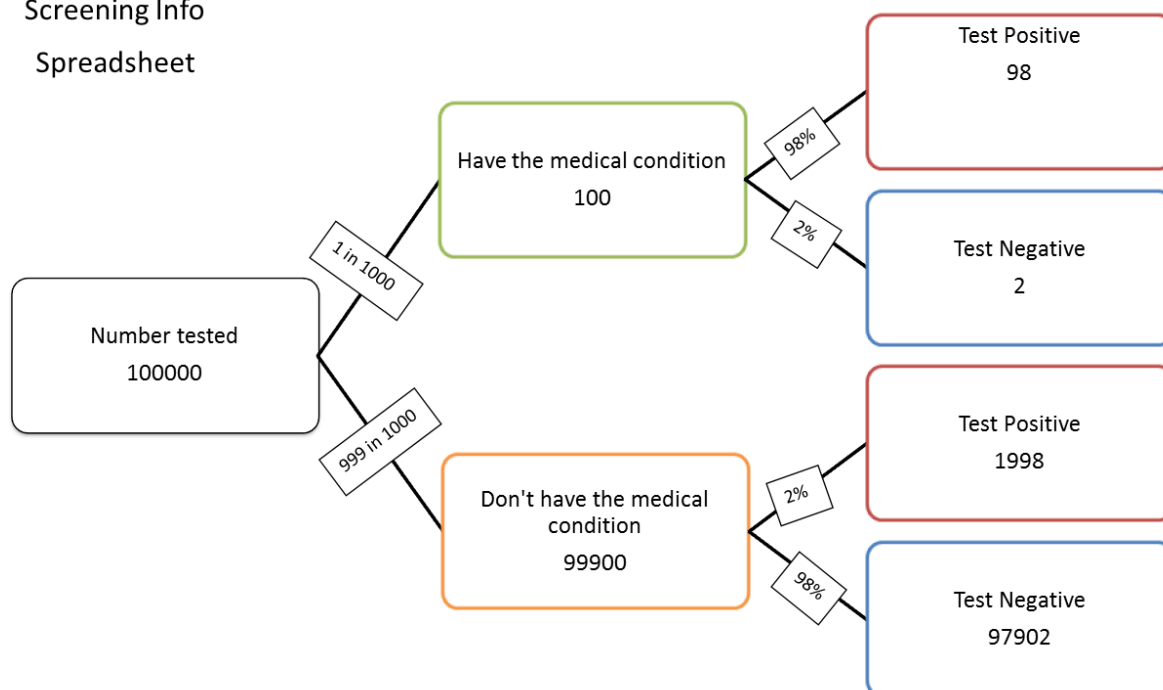


Figure 2 Example solution

There is sufficient flexibility in the teaching materials to allow the teacher to address particular needs, or to make a particular point. For example some teachers may wish to show how this one piece of mathematical reasoning can be used on topics as diverse as email filtering, and the strength of DNA evidence. Other teachers may wish to emphasise how this approach may be useful to the students in situations such as Downs' syndrome screening using real data. Whilst other teachers may wish to explore the structure a little more with questions like:

- Would it make a difference to the probability of a false positive if I reduced the number of people screened from 100000 to 10000 or 1000?
- Why are there so many false positives, given that the test gets it right 98 out of 100 times?

In the trials most students were able to state that the answer is because 98% of 100, is a smaller figure than 2% of 99900. This was often followed by comments about relating to how their understanding of proportional measures had improved.

3.2 Early feedback from the classroom trials

Classroom trials were set up with the aim of experimenting with the teaching materials and the suggested approach. The main method of obtaining feedback was the teacher's impression of the student's response in the lesson. There were reflections from the teacher delivering the session and any observers in the classroom regarding the teaching materials. These mainly took the form of an exchange of emails and verbal comments.

A more robust study measuring the impact of Critical Maths using a teacher and student questionnaire is currently planned. External evaluators from the University of Manchester have produced two questionnaires which were piloted with students and teachers between January and March 2015. There are also plans to produce some in-depth case studies explaining how some schools have used some of the teaching materials.

4 Assessing Critical Maths

4.1 Comparative Judgment

There were some clear differences between the Critical Maths questions and more traditional maths questions. In particular the focus on mathematical process, often relating any calculations back to the context of the question in the form of a reasoned decision. It needed to be very carefully considered

how such questions could be assessed in the context of a written examination. Some industry leading research is being conducted into using the method of comparative judgment in assessing mathematics (Jones and Alcock, 2012). Consequently, a trial for Critical Maths was conducted comparing traditional marking and comparative judgment, (Jones and Wadon, 2014).

4.2 Questions raised regarding assessing students work and examination question structure.

Working with students in the classroom and the assessment trial raised several important questions regarding the assessment and scoring of answers.

In one lesson on estimation students are asked the question “At one time it was claimed that all of the world’s population could fit on the Isle of Wight. In 2011 the world’s population was estimated to be 7 Billion, would this claim still be true?” (Critical Maths Free Resources – Making Decisions from Limited Information)

All of the chosen methods to answer this question required the students to find an estimate of the area of the Island. Some students were quick to approximate the area of the Isle of Wight to a rhombus, or two triangles and then use approximate measurements from the map to estimate the required dimensions of the shape(s) and hence the area of the Isle of Wight.

Other students tried to be more accurate. One student constructed a series of triangles around a point similar to the figure 3.

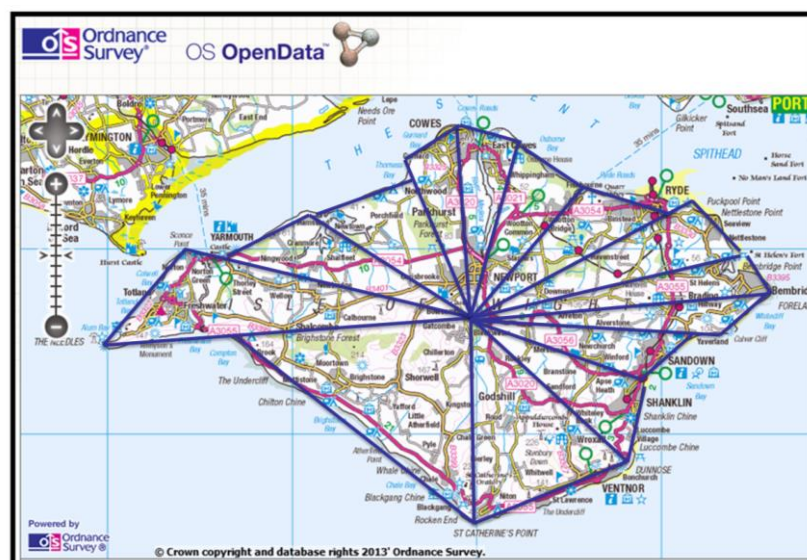


Figure 3 One student’s solution to a ‘mapping’ problem.

When discussing this work with the students, comparing and contrasting the different approaches, the teacher was able to establish that both methods reached the same conclusion which was, no the world’s population would not fit on the Isle of Wight. However the first method, using a rhombus or two triangles, arrived at the conclusion quicker and with less sophisticated calculations.

This poses a question for the assessor, should the method with the less demanding mathematics actually be rewarded with a higher score?

This question prompted a number of interesting discussions at teacher meetings. The conclusion being that any system of assessment should be aimed towards rewarding the desired response. Qualities such as; a clear explanation of any assumptions being made, commentary linked to the calculations to inform the assessor, communication of compromises made in order to achieve a quicker solution and an understanding of how this effected accuracy. The mathematical accuracy of any calculations should also be rewarded.

The questions used in the assessment trial also raised similar issues. In particular how should any assessment be weighted, it is obvious that there should be marks for computational accuracy - but what proportion of the total assessment mark?

The answer to this question depends on the aims of the curriculum. If the aim is to have students that can demonstrate their understanding of how the calculations which they perform are applied to context, then a significant proportion of the marks should be for using their answers to their calculations to explain the implication of these figures in the context. It should also be that in addition to being able to calculate certain statistics, the students should be able to explain the significance of any statistical figures calculated. Finally, as well as calculating and explaining figures, the students should be able to explain why the information they are using is valid, or suggest possible reasons why some information is not necessarily strong evidence.

4.3 Implications for teaching

Following the questions raised in the assessment trials, it is clear that a curriculum based on the ideals of Critical Maths will make different demands from learners. They will need to be able to use the results of their calculations to make considered conclusions about the context being examined. Some questions may not contain all of the necessary information required to produce an answer, in these instances a justifiable estimate could be required, or the acknowledgement that a conclusion cannot be reached without additional information.

Literacy skills will be required to read and comprehend the contextual setting, plus the effective communication of their solutions. Careful consideration will need to be given to developing these skills alongside any mathematics.

In addition to materials produced to support the learning, video footage of Critical Maths lessons have been created in collaboration with experts. These will be made available for teachers to aid and assist them in the delivery of such a curriculum. There are also plans to produce a series of case studies, using a lesson study approach, which should provide evidence of the teacher and student response to Critical Maths.

5 Summary and the future use within Examination Boards' qualifications

In creating Critical Maths there has been a deliberate attempt to produce a mathematics curriculum which would engage a group of students who currently chose to cease studying mathematics when they reach 16 years of age, often with little regret.

The questioning and discussionary disposition of a Critical Maths approach is somewhat different to that of many traditional mathematics curricula. However, indications from trialing the teaching resources suggests that the vast majority of students did find such lessons engaging, but it would be valuable to gather further, more robust, evidence.

It is not possible in this paper to give detail of all of the content created for Critical Maths. However, its use in the immediate future is very encouraging with all the major UK Examination Boards including varying amounts of the Critical Maths curriculum into their new national Core Maths qualifications. Details of just how Critical Maths and Core Maths fit together can be seen on the MEI website: www.mei.org.uk/critical-maths

It is therefore hopefully anticipated that inclusion in these qualifications will offer success in showing the relevance of mathematics to students and overturning some of the negative perceptions held by a significant proportion of learners in this target group.

Critical Maths is an innovative curriculum, designed to cultivate mathematical reasoning and quantitative analysis through questioning, discussion and solving realistic contextual problems. Its widespread inclusion in Core Maths qualifications will enable more students to develop these vital skills and to appreciate the relevance of mathematical techniques to everyday issues.

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