

Hitchin wins Shaw Prize

This year's Shaw Prize in Mathematics was awarded to Professor Nigel Hitchin FRS (Savilian Professor of Geometry, University of Oxford) 'for his far reaching contributions to geometry, representation theory and theoretical physics. The fundamental and elegant concepts and techniques that he has introduced have had wide impact and are of lasting importance.'

Indeed, Hitchin's profound and imaginative work has had tremendous impact in many different areas of Mathematics, including algebraic geometry, differential geometry, complex analysis, topology, integrable systems, mathematical and theoretical physics.

So much so, that to choose which of his many spectacular results should be explained in this article is a daunting task. Last week at a conference in Moscow, I realised that the definition of Nigel's most influential contribution depends on the field of research of your interlocutor.

"Surely it must be the Hitchin's connection!"

"Hang on a second, what about his notion of generalised complex manifold?"

"Nonsense, the self-duality equations are the most important."

"You're forgetting the Hitchin's integrable systems."

"...And the monopoles."

I changed subject, mathematicians can be fiercely opinionated at times!

Hitchin derived the self-duality equations in the context of the self-dual Yang-Mills equations that describe the behaviour of elementary particles in the Euclidean 4-dimensional space. In [1], he discovered that despite the fact that these equations lose all physically relevant solutions when restricted to the plane, they actually produce a surprisingly rich space of solutions when defined on a *compact Riemann surface*, i.e. a surface that can be covered by a finite collection of planes. In the same paper he proved rigorously that solutions of the self-duality equations define what he called *stable pairs* in differential geometry.

This is a typical feature of Hitchin's ground-breaking work: he proves that certain objects arising in theoretical physics (as for example the Higgs fields responsible for the famous Higgs boson) define some new concepts in algebraic or differential geometry. He then develops a rigorous pure mathematical theory of these concepts to deduce very powerful and elegant results that keep the physicists busy for twenty years or so.

Another example of this feature is given by his rigorous approach to geometric quantisation of moduli spaces. In the setting developed by Konstant, Kirillov and Souriau, the first step to quantise the set of smooth real functions on a manifold is to associate to each such function f a first order differential operator F . Next, one needs to choose a *polarisation* (this means that one needs to split the phase-space in position coordinates and momenta and choose a complexification) and finally quantise – there is a very serious difficulty though: proving that the resulting quantisation is independent of the choice of polarisation. Hitchin vanquished this problem by introducing yet again a new mathematical concept: the famous Hitchin connection on the moduli space [3].

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A more recent example is the notion of generalised complex structure [4], which is an analogue of complex structure not on the tangent bundle T of a manifold, but on the direct sum of the tangent and cotangent bundles $T \oplus T^*$. This structure includes many other mathematical structures (such as symplectic structure and Calabi-Yau structure) as special cases and lies at the basis of most recent results in topological string theory.

Hitchin keeps the mathematicians busy too, indeed his celebrated integrable systems [2] provided the basis for the formulation of the geometric Langlands correspondence that in turn has given food for thought to a huge number of pure mathematicians who have dominated all the International Congress of Mathematicians since the nineties.

In the best tradition of longevity established by the founder of the Shaw prize himself, who died at 107 years of age, Nigel Hitchin defies the stereotype that mathematicians need to be in their early

thirties to produce great results. When already in his sixties, Hitchin introduced the notion of co-Higgs bundle [5], an adaptation of Simpson's Higgs bundles in which the tangent bundle is replaced by the cotangent bundle. This inspiring work will surely focus a great deal of research in the future.

On a personal note, Nigel is a lovely man who has served the mathematical community in the UK and overseas by

taking on really heavy responsibilities such as Chairman of the RAE Pure Mathematics Panel (twice!), President of the LMS and many others. He has a wonderful sense of humour and cooks the best pheasant in red wine sauce I ever tasted.

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Acknowledgement: Photograph courtesy of the Archives of the Mathematisches Forschungsinstitut Oberwolfach.

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