

Mathematics Nomad Wins Abel Prize

Give me a museum and I'll fill it.

– Pablo Picasso

Prolific mathematician Yves Meyer, could be described as the Picasso of the mathematical sciences. On 23 May 2017 the French mathematician was awarded the Abel Prize, which is often described as the mathematics Nobel. In the laudatio the Abel recognised Meyer in particular ‘for his pivotal role in the development of mathematical theory of wavelets’.

Wavelets are functions that can explain complex structures in signals and images, in solutions of partial differential equations, by decomposing them into translated and dilated versions of a *mother wavelet* (see Figure 1).

They form an orthonormal basis of square integrable functions and can be seen as a further development of the Fourier transform, characterising signals in time-frequency by spatially localised, somewhat oscillatory, building blocks of different scale and amplitude.

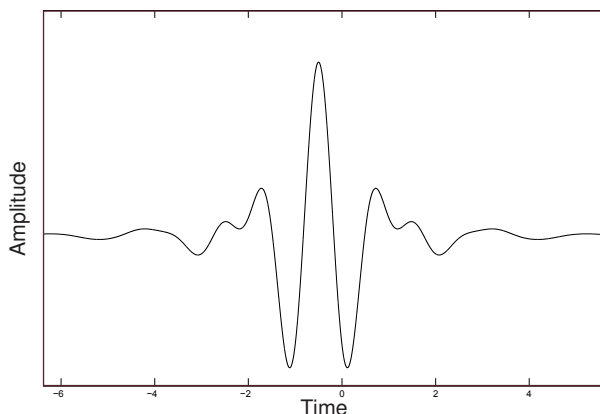


Figure 1: Meyer mother wavelet.

Triggered by his work on singular integral operators (in particular Calderón-Zygmund operator theory) in the 1970s where he encountered the principle of representing an arbitrary function as a combination of functions which are localised in space, Yves Meyer (together with other colleagues such as Ingrid Daubechies and Ronald Coifman) started the wavelet revolution in signal processing in the late 1980s. It all started with a paper by two of the founders of Wavelet theory (Alex Grossmann and Jean Morlet) that fell into Meyer's hands in 1984. Remembering this moment, Yves Meyer described the effect this encounter had on him, saying:

I took the first train to Marseilles, where I met Ingrid Daubechies, Alex Grossmann and Jean Morlet. It was like a fairy tale. I felt I had finally found my home.

In the proceedings of the ICM in 2010 Ingrid Daubechies writes:

Wavelets and wavelet packets are now standard, extremely useful tools in many disciplines; their success is due in large measure to the vision, the insight and the enthusiasm of Yves Meyer.



Indeed, his contributions thereafter crucially changed common practices in signal processing. As an example, the compression standard in JPEG2000 is entirely based on the ability of sparsely representing images in a wavelet basis and the work of Martin Vetterli and his team who turned Meyer's insight into usable computational algorithms. The discovery of wavelets as a tool for sparsely representing images also turned them into one of the central components in compressed sensing, i.e. the non-adaptive compressed acquisition of data. And in 2015 and 2016 wavelets played a central role in the detection of gravitational waves by LIGO. Wavelets separated the gravitational waves from instrumental artefacts and random noise using an algorithm

designed by Sergey Klimenko.

Recent works by Stéphane Mallat and colleagues also show the role of wavelets in understanding the mechanisms behind deep learning. But even beyond signal processing,

Meyer's intuition on the interplay between low and high frequency components of functions led to many important theoretical advances in the fields of harmonic analysis and partial differential equations,

said Terence Tao [1], who gave a presentation about Meyer's work as part of the Abel Prize announcement on 21 March.

Yves Meyer has made several contributions in mathematics, among them in number theory, harmonic analysis and partial differential equations. Indeed, what is remarkable about his work is that it contains both very deep and fundamental mathematical results in a variety of fields with important applications.

In number theory, his work on Pisot numbers, a class of numbers that include the golden ratio $(1 + \sqrt{5})/2$ whose powers lie surprisingly close to integers, has resulted in the definition of sets of such numbers which behave almost periodically. These sets later helped to explain physical properties in so-called quasicrystals, discovered by Dan Shechtman, who was awarded the 2011 Nobel Prize in Chemistry for this work. These crystal structures are mathematically regular but do not repeat themselves, very much like Meyer's sets.

Meyer's work on wavelets also impacted the analysis of partial differential equations, such as explaining (at least partially) the phenomenon of turbulence in fluids by breaking down a particular class of highly oscillatory solutions to the Navier-Stokes equations, partial differential equations that simulate fluid flow, in terms of its different Wavelet coefficients.

Describing himself as a 'nomad', both in his private life and in mathematics, what was always important for him was not to be stuck in a steady state in one topic. Yves Meyer is one of those rare mathematicians who has genuinely crossed the boundaries of several mathematical fields, their theory and application, leaving deep marks in all of the areas he has touched. Developing rigorous mathematical tools close to applications is something many researchers aspire to but also something that is rarely carried out. Yves Meyer, throughout his whole life, has been doing exactly that. He says:

I am not smarter than my more stable colleagues. I have always been a nomad – intellectually and institutionally.

His students and colleagues describe him as a ‘visionary’ (Stéphane Mallat) and his work breaks the frontiers between pure and applied mathematics, computer science, physics and engineering, and as such cannot be categorised but is simply ‘amazing’. Yves Meyer’s work is a model example for curiosity driven research [2] that crosses boundaries between fields and that requires courage and openness with respect to other fields. Meyer claims:

You must dig deeply into your own self in order to do something as difficult as research in mathematics. You need to believe that you possess a treasure hidden in the depths of your mind; a treasure which has to be unveiled.

And when asked recently by the French newspaper *Le Figaro* what advice he gives to young researchers he says:

To disobey. It is necessary to know the tradition but also to question it ... without disobedience, no innovation.

Acknowledgements

The author would like to thank Mila Nikolova from ENS Cachan for her advice when preparing this tribute.

Further Reading

For more on wavelets, the author recommends [3–5]. For more on Yves Meyer’s Abel prize, see www.abelprize.no and [6–7].

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