

Editorial

While on holiday in Italy three years ago, my wife and I visited a magnificent exhibition of arts and crafts by 20th century Spanish painter, sculptor and designer, Salvador Dalí. Alongside his bizarre and incredibly expensive work, I was pleased to spot sculptures of the five regular polyhedra (Figure 1) that demonstrate his interest and ability in mathematics. Dalí also famously depicted a partial dodecahedron in his painting *The Sacrament of the Last Supper* and tackled other mathematical concepts including spheres, spirals, double helices, symmetries, catastrophe theory and the net (unfolded form) of a tesseract (four-dimensional cube).

Such combinations of artistic and mathematical skills have existed for millennia, as scale, composition and perspective rely heavily upon geometry, topology and trigonometry. More recently, the European Renaissance created wonderful legacies in this respect; see [1] and [2] for example. During this period, Italian genius Leonardo da Vinci printed a diagram of the rhombicuboctahedron, while German Albrecht Dürer engraved the truncated triangular trapezohedron and displayed its accompanying net. Indeed, a famous unsolved conjecture about the existence of nets is named after him. Later still, the sketches, tessellations and optical illusions of 20th century Dutch graphical artist M. C. Escher are truly superb. I particularly like his *Hand with Reflecting Sphere*, though he also sketched fabulous drawings of impossible objects such as those described in [3].

The book *Shaping Space* [4] contains 22 interesting chapters on the use of polyhedra in nature and art, which admirably display the enormous imagination and talent that exists. Reading these articles increased my awareness and appreciation of this subject dramatically. I contemplated the regular polyhedra (Platonic solids) and semi-regular polyhedra (Archimedean solids) for a while and pondered about projecting the globe onto an icosahedron. This could then be unwrapped onto a bounded

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planar region to produce a reasonably good approximation that is suitable for two-dimensional mapping of the earth's surface, for which an exact projection has long been known not to exist. This idea is far from new though: Buckminster Fuller devised the Dymaxion map using the same proposition back in 1943.

Amazingly, however, I was delighted to discover the name of a Rattenberg crystal glass ornament that I bought for £10 in 1984 whilst a student on holiday in Austria (Figure 2). It is a convex polyhedron with 80 triangular faces, of which 20 are equilateral and 60 are isosceles. Although uncoloured, it acts as a prism that refracts and reflects light to generate a spectrum of colour. Apparently, it is called a pentakis icosidodecahedron because a kis operator (which replaces a face with a pyramid) is applied to all twelve pentagons in an icosidodecahedron. Which clever glassmaker designed and created that without access to Wikipedia?



Figure 2: Rattenberg crystal glass.

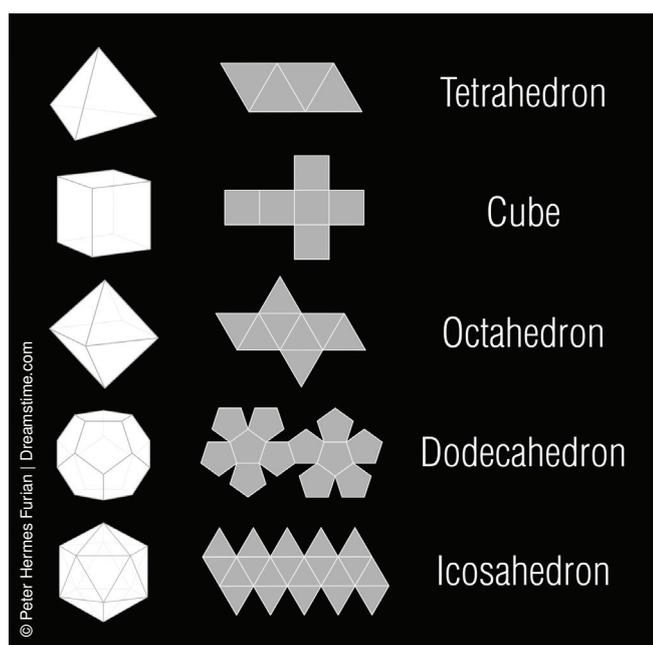


Figure 1: The regular polyhedra.

The strong links between art and maths continue to astonish us with original, awe-inspiring creations. Holograms can display objects in three dimensions and there remains much interest in reducing 3D images to two dimensions. Trompe l'oeil is a classical artistic technique for creating amazing perceptions of depth, which developed in Ancient Greek and Roman civilisations. It now surfaces as stunning pavement art and incredible optical illusions involving floating objects, confusing images and adaptive camouflage. My favourites among these are the hero's leap of faith in the 1989 film *Indiana Jones and the Last Crusade*, and the brilliant car advert that Honda produced in 2013.

A related and equally remarkable development is the long-established architectural equivalent that is forced perspective. This has spawned an internet sensation through taking photographs of friends apparently supporting the Leaning Tower of Pisa, twiddling with the top of the Eiffel Tower, kissing the Great Sphinx or playing basketball with the moon. Yet more impressive, a *Click Art Museum* opened in Chennai last spring and offers many hilarious photo opportunities for snap-happy explorers in India.

Another type of projection involves colour, random-dot autostereograms, which caused a sensation when they appeared in the 1990s. By focusing beyond these 2D images, many people can see 3D shapes without any optical aids. I even have a deck of cards that facilitates mind-blowing, mind-reading demonstrations:

Figure 3 shows the backs of two of these cards. You can also find autostereographic animations, though many more exciting artistic innovations exist. Among these are the skilled technique of anamorphosis, Op art paintings of waves or spirals that appear to be in constant motion, fractal art, photographic mosaics and the guaranteed surprise of reverse perspective. Researching these imaginative art forms is worth a few minutes of anybody's time.

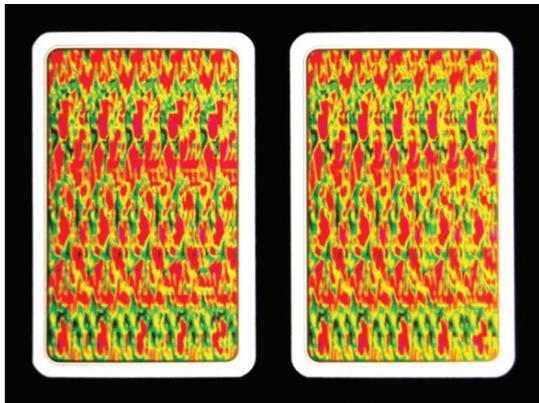


Figure 3: Autostereographic playing cards.

Visiting the same gallery three years earlier, we explored an exhibition that displayed the *Lucan Portrait of Leonardo da Vinci* shortly after its discovery in a private house (Figure 4). The museum's curator gave us a leaflet, which described several scientific tests that had been conducted to ascertain whether this is actually a self-portrait. If it were, then this would be a rare and valuable painting. Among the tests is carbon dating, which gives a probability of 0.64 that the age of the painting is correct. There are also comparisons with authentic art, handwriting, fingerprints and pigments, all of which are fairly convincing in attributing the work to this artist, though whether the painter is also the sitter is less certain. Linear algebra, wavelet transforms, numerical analysis, probability theory and statistics can all benefit art authentication [5].

Such algorithmic verification has much in common with signal and video processing, image analysis and pattern recognition, all of which are so important in matters of defence, security, policing, forensics, industry and healthcare. Closed-circuit television monitoring, automated number plate recognition, radar and sonar analysis, location and trajectory determination, face and speech identification, iris and retina scanning, material matching and speech recognition are a few such applications.



Figure 4: Da Vinci's Lucan portrait.*

Just as the jury is out on the Lucan portrait, other recent authenticity claims have come to light, including whether the play *Double Falsehood* was written by William Shakespeare [6]. Indeed, while the famous *Chandos Portrait* of Shakespeare at the National Portrait Gallery in London is presumed authentic, the *Flower Portrait* owned by the Royal Shakespeare Company was declared a fake following X-ray analysis in 2005.

Immediately after visiting this exhibition, we made a beeline for a gelateria across the road (Figure 5). I mention this now, as my eyes lit up when asked what three flavours I wanted out of the 70 on offer. How many possibilities are there?



Figure 5: Italian ice cream parlour.

As my scoops could replicate flavours, the appropriate formula for the number of combinations with replacement gives $C(72,3) = 59,640$ or $C(73,3) = 62,196$ if we include missing scoops. Wow – I shall have to visit the gallery a few more times to try all those variations, though the cone with three missing scoops might be a bit tasteless!

I trust that you enjoyed reading October's special issue on transport. Having progressed from walking to horses, boats, ships, balloons, bicycles, trains, trams, cars, motorcycles, buses, lorries, submarines, hovercraft, aeroplanes, helicopters, rockets and autonomous vehicles, we can but wonder what marvels the future holds. In contrast, December's issue contains a selection of contributed articles on diverse topics including tennis, drums and cryptanalysis, which I hope you will find equally entertaining.

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* By Leonardo da Vinci – Photo by Nico Barbatelli, Public Domain, <https://commons.wikimedia.org/w/index.php?curid=12107718>