

Cultural Challenges Facing the Mathematical Sciences

It is a great privilege to serve as president of the IMA. The Institute has a vital role to play in supporting the advancement of mathematical knowledge and its applications, and it is clearly an honour to be asked to play a leading part in this endeavour. A great tradition of the IMA is that the president visits branches around the country to deliver their Presidential Address but that puts quite a bit of pressure on the choice of topic! I decided that the most appropriate course of action would be to speak about how

I see the challenges facing the discipline as someone who has in the past led a mathematics department but who now is in a more senior university leadership role. Hopefully, having seen things from both sides of the fence, I can make a valuable contribution.

My first observation is that when I refer to mathematics or the mathematical sciences I do so in the broadest sense. I include 'pure' mathematics, 'applied' mathematics (though I dislike the distinction), statistics, operational research, mathematics education. I include research, teaching and training. The subject I am concerned with is not the preserve of a few individuals pushing back the frontiers of knowledge, it is pervasive: in universities, schools and colleges; in business and industry; in everyday life. My focus here is on things that we as a community can do to create a better future.

Let's start with a claim that is probably self-evident to you. Mathematics matters. Mathematics has mattered for an awfully long time. It mattered 2000 years ago when people were surveying land or constructing accurate calendars. Its importance rocketed in the 17th century with the realisation, following Galileo and Descartes, that mathematics is the language of science. Not only is mathematics useful, everything eventually comes down to mathematics. The whole of the physical sciences and engineering is underpinned by mathematics – you only have to examine undergraduate curricula to see that – and by the end of the 21st century we may be saying the same thing about the life sciences.

Today, mathematics matters more than ever. To quote from the 2012 Deloitte report *Measuring the Economic Benefits of Mathematical Science Research in the UK*:

Without research and training in mathematics, there would be no engineering, economics or computer science; no smart phones, MRI scanners, bank accounts or PIN numbers [1, p. 5].

The report's findings include that around 10% of the UK workforce (i.e. 2.8 million people) are in occupations which either entail mathematical science research or which use mathematical science research-driven tools and techniques. The economic value of mathematical sciences research was estimated at £208 billion in 2010 (16% of the UK total GVA). The most significant sectors of the economy that benefit from mathematics include Finance, IT, Defence, Pharmaceuticals, Construction, Health and Education. So that's pretty much everything!

It is not just mathematics research that matters. We need an education system that both produces people with the appropriate knowledge and skills to fill the jobs that will be in demand over the coming years (a moving target which is difficult to predict) and provides the opportunities for the next generation of mathematics researchers to emerge.

... This begs the question: are our undergraduate curricula really suitable for the 21st century? ...

There has been quite a bit of effort mapping this people pipeline in recent years. I would question, however, whether the knowledge we have gained has been turned into much action.

If we look at data related to PhD students [2, p. 14] then one thing is obvious: the vast majority of PhD graduates do not become academics. This is well understood, but I think our actions sometimes appear as if we only really care about those who do make that progression. There is a tendency within mathematics to see those students that end up as PhD students as the successful ones. Or those PhD students who end up as post-docs as the successful ones. But an undergraduate who walks straight into a graduate level job in the finance sector has succeeded by any measure. Moreover, they will have done it because of their mathematics degree not in spite of it. Such students and others who make different career choices should be seen as success stories by the profession, treated as such, and encouraged to form part of the professional mathematics community.

It is also clear that we need a lot of new PhD students every year if we are to fill the academic jobs of the future. But the education that we provide these students must take account of the fact that most will end up in careers outside academia. A recent EPSRC People Pipeline report [3] posed the question: *Should a Mathematical Sciences PhD create a world class mathematician or someone suited to a range of employment options?* Providing an opportunity which enables both to happen is of course ideal.

There have been many reports written about mathematics over the past twenty years or so. In terms of looking forward, I would recommend the 2013 report of the US National Research Council, *The Mathematical Sciences in 2025* [4] (freely available at <https://www.nap.edu/catalog/15269/the-mathematical-sciences-in-2025>). The authors argue that the mathematical sciences enterprise in the early 21st century is qualitatively different to that of the latter part of the 20th.

One of the report's key observations is that the value of the mathematical sciences would be heightened if the number of mathematical scientists who share the following characteristics could be increased:

- They are knowledgeable across a broad range of the discipline, beyond their own area(s) of expertise;
- They communicate well with researchers in other disciplines;
- They understand the role of the mathematical sciences in the wider world of science, engineering, medicine, defence, and business; and
- They have some experience with computation.

Of course this will only happen if they are exposed to these things. This begs the question: are our undergraduate curricula really suitable for the 21st century? In June I presented my address at GCHQ and was struck by the clear message I got from the audience: mathematics undergraduates do not get enough exposure to computation. What GCHQ needs are people who can develop new mathematical ideas and then put them into action using computers. I'm sure there are many other employers who would tell a similar story.

Actually, much of this has to start at school and the lack of

qualified mathematics teachers (some estimates put the shortfall at over 5000) makes this a real challenge. Many initiatives have been tried over the years to help rectify the problem, some generic to the teaching profession, some specific to mathematics, and there is a welcome focus on mathematics in current government plans. The IMA is also doing its bit. We run the Maths Teacher Training Scholarship scheme on behalf of the DfE and successfully recruited over 250 new talented trainee maths teachers into the profession for this academic year.

But what do the providers of mathematics degree programmes do to encourage the brightest and best to pursue teaching as a career? How many mathematics degrees include courses designed to engage students with the fundamentals of mathematics education? Given the obvious long-term benefits of having a sufficient and sustainable number of high-quality mathematics teachers in our schools, do we do enough to promote teaching as a profession to undergraduate mathematics students? My guess is that we could do a lot more.

Returning to the importance of mathematics, I think we should recognise that the importance of mathematics in the 21st century is due to both new applications of old mathematics as well as the impact of new mathematics. It is tempting when arguing for more funding for mathematics research to emphasise the benefits that such research may have in the future. This thinking is reinforced by the need to describe possible pathways to impact in grant proposals. However, the best evidence for the future benefits of mathematics research is the benefits we are getting now from past mathematics research. And because mathematics is never past its sell-by-date some of these benefits are the result of mathematics research done a long time ago.

Extracting value from mathematics requires people making connections between mathematics research and applications. This is a skill which is as important a part of the impact generation process as the creation of the mathematics in the first place. The mathematics research community should attach much more value to those individuals who have these skills than is currently the case. Apart from the obvious benefits of getting more out of old mathematics as a result, and helping to solve important contemporary problems, this will lead to a greater body of evidence with which to argue for greater mathematics funding in the future.

As an example, consider PageRank, the algorithm used by Google to rank its search engine results. This was developed by Larry Page and Sergey Brin at Stanford University in the 1990s, building on work from the 1970s which looked at ways to rank journals based on citation data. It required insight into information contained within directed graphs and then turning this into an eigenvalue problem which can be solved using an iterative process. The PageRank algorithm, which has been heavily refined since its original inception, brings together mathematical ideas from linear algebra, graph theory and probability theory which were initially developed in the 19th century, and combines them to solve a problem which did not exist until very recently. It led to the creation of Google and over US\$300 million for Stanford. It has had a profound impact on the lives of billions of people.

PageRank should be seen as a triumph for mathematics. It's a triumph for computer science too; they are not incompatible. What we must not do is let the fact that the mathematics involved was not particularly new lessen our appreciation for the mathematical skill required to create such a game-changing device.

Every pupil studying maths at school should be taught about PageRank. An internet of 3 or 4 sites makes the maths accessible

at A-level. This begs the question: how many teachers, including those educated before PageRank was created, know enough about it to teach it? Taking that a bit further, how many mathematics undergraduates learn about PageRank? My guess is not many, certainly not enough. This is not just a pity; it is a profound mistake as these students will not be able to pass on the fact that PageRank is mathematics to those they go on to inspire.

It is an example of a broader problem. Not enough effort is spent on giving mathematics undergraduates a broad and, critically, up-to-date understanding of the expansive reach of the mathematical sciences. If mathematics graduates don't know what mathematics is being used for in today's world, who is going to spread the knowledge to funders, policy makers, schoolchildren?

One of the primary causes of the massive increase in reach of the mathematical sciences over the past few decades has been the exponential rise of computing power (see Figure 1). The pervasive nature and power of computer technology has transformed the world.

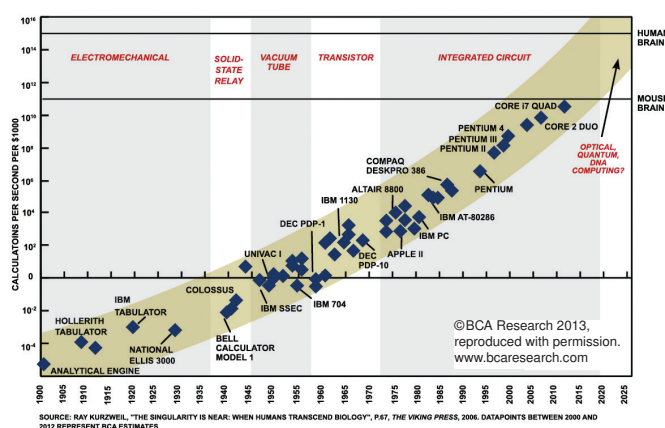


Figure 1: The exponential rise of computing power.

There are many areas of mathematics where development largely stopped when researchers hit a wall, with the calculations simply too lengthy to do by hand. Computers have changed all that. On the one hand this has meant that we can now apply mathematics where previously there was little value and on the other, there are areas of mathematics that have been reinvigorated.

There seems no reason to believe that this link between the growth in computing power and the reach of the mathematical sciences is going to break any time soon. We need to own this mathematics, much of which is not new, and we need to tell our undergraduates about this expanding reach so that they can tell others. We also need to recognise the change in what is driving the discipline and that involves exploring much more than might traditionally be regarded as core mathematical activities.

Mathematics 2025 [4] emphasises the expanding reach of the mathematical sciences. Examples given include: social science networks; protein folding; climate modelling; computational biology; artificial intelligence; public health; metamaterials; and compressed sensing. Now these topics are not branches of mathematics. They are areas where mathematics (interpreted broadly) has a significant role to play in helping to address important issues. There is also a feedback loop in that the problems that are out there in search of a solution drive change in the environment within which mathematics is done and hence, both directly and indirectly, influence the development of mathematics.

Currently, there is a bit of a disconnect between the messages we seek to promote around the importance of mathematics in a

growing array of activities with significant economic, social and cultural benefit, and the idea that the developments in these other areas of investigation should impact on how the mathematical sciences evolves. For example, it seems entirely reasonable to me that there should be an expectation of a qualitative shift in the proportion of mathematics research undertaken which is relevant to data science as a result of the explosion of applications that is likely to materialise. We should also be teaching more data-related mathematics to our undergraduates.

There are a number of universities that offer data science degrees. Mostly these are essentially computer science degrees. As far as I can tell there are only three universities that offer a data science degree that is heavily mathematical: Aberystwyth, Nottingham and Warwick. Not long ago there was only Warwick and I expect to see more enter the market. It is important that this happens because we need to ensure that young people see the relationship between mathematics and the data revolution.

A constant refrain from the mathematics community is that there is not enough money available to support mathematics research. I believe that this is true. There is certainly ample evidence that spending money on mathematics research will pay significant long-term dividends. But while making the case for the importance of mathematics is certainly a good thing in its own right, the real opportunity here is not to get a bit more funding for core mathematics research, but to extend the reach of the mathematical sciences research base so that many more agencies

see the mathematical sciences as a natural home for their funding. All the UK research councils should see mathematics research as an important part of their portfolio. Mathematicians should be leading multi-disciplinary projects spanning research fields aimed at tackling the big issues of the day. Often the role of the mathematician is to find the links between existing mathematics and the application. This is still mathematics research.

This is not a suggestion that all our energy should be focused around generating money to study the applications of mathematics. We must support the whole ecosystem. The fact that there are so many opportunities to apply mathematics is due to the strength of fundamental mathematics research; research that was embarked on with no application in mind, simply to enrich our understanding. One of the consequences of extending the reach of mathematical sciences must be to channel some of the extra money that is drawn into the subject to grow the core.

It is interesting to reflect on what we should think of as the mathematical sciences community, a phrase that is often used (by me, for example!). *Mathematics 2025* suggests that the mathematical sciences community is the collection of people who are advancing the mathematical sciences discipline and this includes a great many people who are aligned professionally with more than one subject area.

The pace of change in the external environment is, unsurprisingly, not matched by changes in the way we organise ourselves and as a result the mathematical sciences now extend far beyond the academic departments, funding sources, professional societies, and principal journals that support the heart of the field. This is a powerful message with profound implications. We should embrace it.

One of the problems with the all-encompassing nature of this definition is that people prefer to focus on the specific bit within mathematics that they hold most dear for fear that it will

not otherwise get the attention it deserves. They leave the rest to ‘other people’. The problem is that a landscape full of ardent factions is not one that anyone can draw on to make a cohesive case for the whole. We are much weaker as a community if we organise ourselves into many small groups each of which fights its own corner without any sense of what is in the best interests of the whole. The split between pure and applied mathematics that still pervades the UK landscape is a case in point.

The concept of an applied mathematician, at least as the term is understood in the UK, seems to have arisen in the mid-19th century. Many of the great mathematicians of the 18th century, such as Euler and Laplace, were equally adept at solving problems relevant to real-world phenomena as making fundamental mathematical advances. Gauss (1777–1855), a name synonymous with mathematical brilliance, could flit between the incredible subtlety of number theory and the practical problem of determining the orbit of an asteroid or comet from a small number of observations.

But after Gauss, such examples are harder to find. Most of the great European mathematicians of the late 19th century took their inspiration from within mathematics itself, but in Victorian Britain a new type of mathematician emerged epitomised by

men such as Airy, Stokes, Thomson (Lord Kelvin) and Maxwell, who were driven by a desire to better understand the world around them. They made huge strides in the development of mathematical theories for a wide variety of physical phenomena, such as celestial mechanics, optics, hydrody-

namics, elasticity, thermodynamics and electromagnetism. The Victorian ethos of utility ran counter to the main thrust of pure mathematics and there appears to have been a widespread feeling in Britain that those who immersed themselves in the abstract world of, for example, group theory or projective geometry, were wasting their talent.

The first half of the 20th century saw a trend towards greater abstraction. Writing in 1941, Richard Courant (an ‘applied’ mathematician) and Herbert Robbins (a ‘pure’ mathematician) expressed some concern in the introduction to the classic *What is Mathematics?*

Once more the pendulum swung toward the side of logical purity and abstraction. At present we still seem to be in this period, although it is to be hoped that the resulting unfortunate separation between pure mathematics and the vital applications ... will be followed by an era of closer unity [5, p. xviii].

They highlight two critical benefits that such a union would bring, ‘regained internal strength’ and ‘enormous simplification on the basis of clearer comprehension’.

In this ultra-competitive 21st century, it seems to me more important than ever for mathematicians to nurture this relationship between the more theoretical and more applied aspects of the discipline. Let me quote Kolmogorov (and thanks to my Loughborough colleague Karima Khusnutdinova for the translation):

Mathematics is vast. One person is unable to study all its branches. In this sense specialisation is inevitable. But at the same time mathematics is a unified science. More and more links appear between its areas ... Therefore an isolation of mathematicians in too narrow borders would be destructive for our science [6].

... All the UK research councils should see mathematics research as an important part of their portfolio ...

Many mathematics departments have separate pure and applied groups and this has the effect of creating artificial barriers. What typically determines which side of the fence you are on is the field you work in. If you are a number theorist, you are ‘pure’ though you may well be interested in applications, e.g. cryptography. If you study PDEs arising from the theory of elasticity you are ‘applied’ though your work may be a million miles from any application. Differences in culture (which certainly exist) are accentuated and opportunities for the exchange of ideas no doubt missed. As you get used to the environment in which you operate, and if you are not exposed to the other world, the effect is that it all remains a mystery simply because you can’t engage with it at an appropriate level. Put simply, mathematicians need to work together.

It is interesting to observe how mathematics in the 19th century is described by writers today. More often than not, historians of mathematics ignore applied mathematics (except in work specifically about applied mathematics). This comes from the modern tendency (in my view mistaken) to undervalue the development of links between mathematics and other disciplines, or at least not to think of doing this as mathematics.

There are other modern value judgments that I think are problematic. Here are some things that I believe are over-valued in academia:

- Who your PhD supervisor was
- Willingness to put mathematics first and family second
- Highly technical results that only three people understand
- Reputations (what you did 10 years ago)
- Chalk

And under-valued:

- Ability to communicate mathematics to others (in and outside mathematics, students, spoken and written)
- Making connections
- Breadth of knowledge
- Leadership

I will leave it to you to reflect on whether you think any of these things are relevant to the appalling under-representation of women in senior academic roles in mathematics.

The final topic that I would like to discuss is *leadership* and I will again use history to illustrate my remarks.

One of the reasons that Poincaré’s ideas did not take off immediately after his death was that he did not build up a group around him to develop the ideas that he was generating. This was very much the culture in France at the time. Pretty much none of Poincaré’s contemporaries in France, men such as Jordan, Picard and Hermite, saw it as their job to develop the next generation of talent to take forward their ideas. Poincaré’s nephew Pierre Boutroux observed of his uncle that:

He did not believe that oral communication, the verbal exchange of ideas, could help discovery [7, p. 203].

The culture in Germany was very different. Klein, Lie and Hilbert all worked in an environment where doctoral students were ever-present and their influence on mathematical developments in the first half of the 20th century was immeasurably greater. This model gradually spread around the world and I would argue that leadership *within* mathematics is alive and well.

But mathematics does not exist in a vacuum. Mathematics departments are part of Schools or Faculties which in turn are

Deaths in the Hospitals of the Army in the East

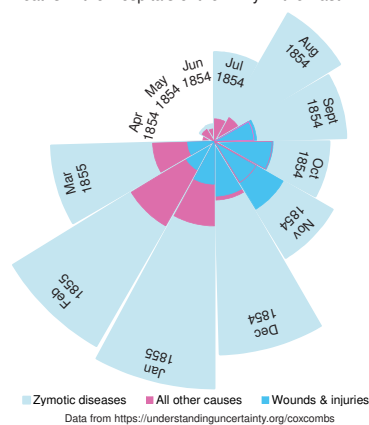


Figure 2: Florence Nightingale and her coxcomb.

part of universities. For the subject to thrive we need people to represent and advocate for mathematics within these structures.

Florence Nightingale didn’t simply identify the fact that poor sanitation was the main reason so many soldiers died in hospital. She didn’t just devise new ways of displaying the statistical data to clearly illustrate the point, she also made sure that those who had the power to change these things knew the facts and she persuaded them to invest in the solutions.

If you want to have that sort of influence you have to work at it. You have to engage with people whose interests do not coincide with yours, who see the world differently and who have a different set of values and priorities. You have to learn their language. Not everyone is cut out for this, nor need they be. The problems arise when there are not enough people interested in taking on this type of leadership role and the responsibilities that go with it.

I would like to hope that a significant number of mathematicians starting out in their careers now see the opportunity to exert influence on behalf of their discipline as something which will be rewarding and worth aspiring to. My own experience is that mathematicians do not value this sufficiently. There is a reluctance to engage with people who do not understand mathematics. This is something that needs to change. I cannot overstate the value to mathematics of being able to influence people who are not mathematicians.

Chris Linton CMath FIMA

IMA President and Loughborough University

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