Editorial

If I were a pair of hiking boots then I could charitably be described as 'worn in'. If I were a piece of clothing then I'd definitely be described as 'pre-loved'. And, regardless of the degree of poetic license allowed in online dating adverts, I really couldn't pass as 'youthful'!

So, I'm now old enough to have seen one or two things change during my life. Don't worry, though, this isn't an editorial pining for 'the good old days', as viewed through rose-tinted glasses. Many of the changes I have seen have been for the better, although, as the Black Lives Matter movement has shown, there are still many more changes that need to be made. That notion of 'we've made some progress, but more is still needed' is probably a theme that has applied, and will continue to apply, throughout all of human history. I'd certainly suggest that it applies to one of the themes of this editorial: bullying.

When I was at school, bullying was one of those things that just seemed to happen, like homework or PE lessons. Unless something dramatic occurred then, at least from my perspective as a pupil, it was swept under the carpet, accepted as part of life. Thankfully, much greater attention is now paid to bullying, whether it occurs in school or in the workplace. But, if we think of bullying in terms of the inappropriate use of strength to subjugate then there are wider aspects that can, and should, be considered.

Some of these were raised by Adnan Darwiche, who talked about the artificial intelligence (AI) community being 'bullied by the success' of machine learning (ML) [1]. More specifically, the significant progress that is being made in ML, whether this is viewed from the perspective of the number of academic papers, or the amount of venture capital raised by start-up companies, is effectively curtailing work in all other areas of AI. You could also argue that general ML is, to a certain extent, being bullied by the success of deep learning. Whilst understandable, this focus on short-term success may be detrimental to the long-term health of the subject and the community.

Of course, there is no reason why the idea of being bullied by success should be restricted to AI. It might apply to large areas of research, both historically and currently. There's probably an interesting discussion to be had about whether the way we measure academic output reinforces this bullying behaviour, or mitigates against it. Unfortunately, to navigate that argument you need someone more talented, and more knowledgeable, than me.

What I might be able to talk about (although even here I am on shaky ground) is how 'bullied by success' or, perhaps, 'bullied by history', which might or might not be the same thing, affects how we prove things and, consequently, what we choose to prove.

I often write, and talk, about the beauty of mathematics. The irony of this is not lost on me: emphasising the subjective, intangible attractions of a logical, and ultimately practical, subject. I guess I'm trying to communicate my passion with words that resonate for many people. For me, part of the beauty is in the way theorems and proofs can encapsulate big ideas in small, apparently perfectly-formed packages. Fermat's last theorem is one obvious example; the four colour theorem is another.

I think these two cases make an interesting comparison. Fermat's last theorem, which states that there are no positive integers a, b and c that satisfy $a^n + b^n = c^n$, for integer $n > 2$, was famously proved by Andrew Wiles [2]. This proof was delivered in the traditional way, via published papers, that extended and developed mathematical approaches. It was, in my opinion, quite rightly hailed as an incredible achievement.

© Viktor Wallon-Hars | iStockphoto

Pavement painted according to the four colour theorem

Colloquially, the four colour theorem states that any planar map can be coloured using only four colours such that no two neighbouring regions share the same colour. Like Fermat's last theorem, the four colour theorem can be easily explained to the general public.

However, unlike Fermat's last theorem, the original proof of the four colour theorem was highly controversial. The approach adopted by Appel and Haken involved checking almost two thousand individual cases, something that was achieved by use of a computer [3]. Over time, simpler proofs have been provided, but the discussions about the nature, rather than the content, of Appel and Haken's proof have always interested me.

… Like Fermat's last theorem, the four colour theorem can be easily explained …

One of the things I really love about mathematics, and one of the things that distinguishes our subject from, say, physics or chemistry, is the notion of proof. A mathematical proof is cast iron; it is rock solid; it can be relied upon. This contrasts with, say, physics, where our understanding is the best one we have that is consistent with experimental evidence. That doesn't mean scientific explanations are useless: Newton's equations of motion are perfectly adequate for an incredible number of applications, for example. They do, however, lack the certainty, and perhaps the purity, associated with mathematical proof.

That background may provide an indication as to why the computer-aided proof of the four colour theorem was so hotly debated. Relying on a computer brings another entity into scope. There's now something involved that, since it is based on physical principles, we don't fully understand, at least in the sense of mathematical proof. Arguably, this changes the nature of proof.

Conversely, some mathematicians believe proofs in the traditional style (including that of Fermat's last theorem) are now too large and complex to be checked by humans. This leads to the notion of using computers to check proofs. Although this does not seem to be widely accepted, there are cases where undergraduates

are being taught this skill, for example via the Xena project (xenaproject.wordpress.com/what-is-the-xena-project).

Considering these two cases, I wonder if the controversy surrounding the proof of the four colour theorem might be indicative of a wider, and potentially damaging, trend. It could be the case that, over time, we have developed an ingrained, cultural preference for things that are, in some generally accepted sense, mathematically beautiful. By that, I mean things that intuitively feel right, in the same way that an integral does after the correct substitution has been made. Equivalently, I wonder if a general preference for traditional proofs, and the sort of thing that can be elegantly proved using them is, in effect, bullying other potential approaches to mathematics.

I'm not suggesting a whole scale move away from the traditional approach to proofs. But, I am very, very tentatively suggesting that beauty may be in the eye of the beholder and that a collective (and overwhelming) preference for things that are beautifully formed in the classical sense may be biasing us away from some aspects of mathematics. In particular, I'm wondering whether we're bullying ourselves away from engaging with things that could be important and, in their own way, beautiful.

Rob Ashmore CMath CSci FIMA
Defence Science and Technology Laboratory

References

1 Darwiche, A. (2018) Human-level intelligence or animal-like abilities?, *Communications of the ACM*, vol. 61, no. 10, pp. 56–67.

2 Wiles, A. (1995) Modular elliptic curves and Fermat's last theorem, *Annals of Mathematics*, vol. 141, no. 3, pp. 443–551.

3 Appel, K. and Haken, W. (1976) Every planar map is four colorable, *Bulletin of the American Mathematical Society*, vol. 82, no. 5, pp. 711–712.