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CONFERENCE ABSTRACT BOOK

Table of Contents

Decumulation of pension savings: pooled annuity funds as a better, greener alternative

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Most pension savers in the UK and elsewhere are saving for their pension via defined contribution pension plans. Yet such plans do not provide an income in retirement. Individuals must decide how to convert their accumulated pension savings into a retirement income. As the two main choices for doing so either are not appealing in the marketplace or are too risky for the average retiree, consideration of alternative choices is justified.

Pooled annuity funds offer a halfway house between life annuities and income drawdown. They allow participants to gain a significantly higher and lifelong retirement income compared to income drawdown, while avoiding the costly guarantees inherent in life annuities.

Pooled annuity funds are more likely to finance green growth initiatives, due to the collectivisation of assets and a common investment strategy, compared to income drawdown. As pooled annuity funds offer few, if any, guarantees compared to life annuities and thus have a less constrained investment strategy, they also have a stronger ability to invest in green initiatives compared to life annuities.

An outline of the workings of a pooled annuity fund, with recent results on their ability to deliver a lifelong income are presented.

Some of the work presented is joint work with Thomas Bernhardt, University of Manchester.

Carbon Markets and Financial Instruments – The jigsaw puzzle of numbers and Climate Change Resolve

Dr. Prachi Ugle Pimpalkhute

ESDW 2021-European Wide Initiative Organizer, Lead and Mentor for Mission De Facto Cities and Communities, Founder Climate Change Crusader Eco Endeavourers Network, IUCN Union Member Commission Environmental Management, SP9P Sustainable Procurement Pledge Ambassador, UN SDGs Ambassador, ACT4SDGs Ambassador, Campaigner & Champion, Association of Commonwealth Universities Higher Education and the SDGs Network, UN Partnership for SDGs Platform Member

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Abstract:

The impending risks to carbon markets and its associated entities and stakeholders has brought into forefront key financial loop holes and gaps. The paucity in uniform, place based, sector-based framework and tools to resolve the underlying issues make it further matter of concern as Chart of Accounts (CoA) in the financial markets do not most often have aligning and applicable principles to strengthen fiscal space of carbon markets, carbon neutrality bonds and Net Zero targets. The carbon markets is a jigsaw puzzle of carbon pricing, 1.5^0 c agreement, shadow pricing (future emissions- scenario), internal pricing (current prices), internal pricing and tax (current pricing), site-specific emission factors for calculation for optimized emission reduction, Scenario calculation where in emission factor of 0.2 EF means $\sum CO_2$ X CH_4 X SO_X X NO_X X NH_3 X 0_3 X CFCs (Without project) – Baseline $\sum CO_2$ X CH_4 X SO_X X NO $_X$ X NH_3 X 0_3 X CFCs (With project). The carbon market is faced with transitional and physical risks with need to derisk portfolio to avoid stranded assets. There is increased carbon markets dependency on GHGs emissions – where in Total Cost = Required GHG Reduction Amount *Price per tCO2e, 3°C Scenario - NDC through the Paris Agreement, 2°C Scenario - Amplification of 3°C and 1.5°C Scenario - Carbon neutrality by 2050. Entity level calculation include : Reserves x carbon factor = CO2 potential; Exchange – level = Sum of company CO2 = Exchange total and Global- level = Sum of exchange totals > Global carbon budget; cost/benefit analysis through levelized cost of energy (LCOE). Varied economic instruments are placing monetary value on carbon - quantitative easing, green bonds, carbon cap and trade, impact investing, ETS and hedge funds to support GHG emission reductions, decarbonization and divestment. The deliverables from the paper include: countries and jurisdictions should leverage existing monitoring, reporting, evaluation systems and other administrative processes to reduce costs associated with carbon taxes as with measurement, reporting and verification (MRV) systems. Identifying outliers, where risks are concentrated to better inform and facilitate carbon reduction strategies and scaling-up market mechanisms explores essential elements of market “readiness”.

Keywords: Carbon markets, Charts of Accounts (CoA), Neutrality, Net Zero, NDC, MRV

IFRS 9 impairment: assessment of significant increases in credit risk using a survival analysis framework

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Since the 2008 financial crisis, regulators focused on ensuring adequate capital provisions are available for banks and other financial institutions to account for a potential increase in credit risk (Basel III and EU CRR regulations). With the introduction of IFRS9, banks are required to estimate expected credit losses (ECL) for their credit portfolios. This is typically done by computing Probability of Default (PD), Exposure at Default (EAD), Loss Given Default (LGD) and applying a forward-looking adjustment.

Expected credit losses are normally calculated on a 12-months performance window for performing loans (stage 1). However, loans showing a Significant Increase in Credit Risk (SICR) are considered as IFRS9 stage 2 and banks are called to compute lifetime expected credit losses. For the computation of lifetime expected losses, Advanzia implemented a survival analysis model framework for the estimation of PD and LGD.

For assessing SICR, banks would need to compare current expected PD with expected PD at origination. According to European Banking Authority, for most banks a loan whose current PD is more than three times its PD at origination would trigger a SICR.
This poses a problem for high-risk loans, since the threshold for SICR could be over 100%. On the other hand, a naïve implementation of statistical significance poses other type of problems.

In order to implement a robust definition of statistical significance for SICR, it is necessary to properly compute prediction errors for both the current PD and PD at origination. The former presents a particular challenge due to the use of survival models for its calculation.
Furthermore, the computation of statistical significance for the difference of two distributions was required, which are non-iid and bounded on both sides.
Finally, the derived definition needs to be robust over time and produced in an automated fashion with minimal interference.

In this paper we propose a potential solution we identified in Advanzia, following an approach that works for higher risk loans as well.

Title: The impact of climate transition risks on financial stability. A systemic risk approach

Authors and Affiliations:
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Intended speaker: Javier Ojea-Ferreiro

Abstract:

In this research we address how climate transition risk, through its effects on asset prices, could impact financial stability. To that end, we characterize the behaviour of financial firm returns conditional on the dynamics of market returns for green, neutral, and brown assets, reflecting low, neutral, and high vulnerability, respectively, to transition to a low-carbon economy. We consider three climate transition scenarios coherent with the NFGS narrative: disorderly transition, hot house world and orderly transition, featured in terms of relative changes in green, neutral, and brown return. We then assess the systemic risk impact of those scenarios on financial firms in terms of the average return (climate transition expected return), the minimum returns with some confidence level (climate transition value-at-risk), and the average return below that minimum threshold (climate transition expected shortfall).
For European financial firms (banks, insurance companies, financial services companies, and real estate firms) over the period 2013-2020, we find that the systemic impact of climate transition scenarios differs widely across financial institutions. Banks experience a greater systemic impact in a disorderly transition than in a hot house world scenario, while the opposite occurs for the other financial subsectors, especially for real estate firms. We also find that the systemic impact of the different climate transition scenarios broadly diverges within financial firm groups (mainly within the bank group), yielding potential winners and losers, and we furthermore study to what extent the systemic impact on financial systems varies across Europe.
We assess the implications of climate-related systemic risk in terms of capital shortfalls. For banks, capital shortfalls are negligible in the orderly transition scenario; however, in the disorderly transition and hot house world scenarios, capital shortfalls are sizeable and concentrated in a small number of entities, although those capital shortfalls can be absorbed within the banking sector. For the remaining financial firms, we find that insurance firms experience small capital shortfalls in any climate transition risk scenario, whereas financial services and real estate firms experience modest capital losses in a hot house world scenario, but negligible capital losses in the remaining scenarios.
Finally, a forward-looking simulation of prospective climate transition measures for the upcoming five-year period suggests that banks may be at a significant disadvantage in a disorderly transition scenario; financial services and real estate firms are likely to experience significant systemic risk effects in the hot house world scenario; and the systemic risk impacts for insurance firms are moderate in size and similar across the disorderly and orderly climate transition scenarios.

Climate transition risk in credit risk determination: evidence from listed firms on the Spain Stock Market Index

Title intended for oral presentation

Authors:

Daniel Ramos-García[a], Carmen López-Martín[a] and Raquel Arguedas-Sanz[a]

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Intended Speaker:

Daniel Ramos-García

Abstract

This work assesses the impact of climate transition factors in the credit risk of Spanish listed companies before and after the signing of the Paris Agreement, through the use of relevant credit factors. For this purpose, the starting point is a Fama-French based factorial model over a panel data sample, from 2010 to 2019. This model is subsequently modified to capture climate impact by means of different functional forms.

In order to do so, a credit risk measure based on the market Merton's distance to default is examined for the main listed companies of the Spain's principal stock exchange in relation to a different performance and sectoral variables, as well as distinct transition risk factors based on the EU-ETS trading scheme information.

This paper contributes to the analysis of transition risk within credit risk using a model based on market perspectives rather than through the use of carbon emissions, which allows for the incorporation of market expectations over alternative technologies or about asymmetries in the implementation of new regulations as well as differences in company mitigation capacity or changes in consumer preferences.

Title: Climate change science, extreme weather and mortality

Type: Oral presentation

Authors: Abdal Chaudhry (Barnett Waddingham), Michael Leitschkis (Milliman), Han Li (University of Melbourne), Qihe Tang (UNSW Sydney).

Intended speaker: Abdal Chaudhry and Michael Leitschkis

Abstract:

Climate risk has recently been at the forefront of insurer's agendas. Yet, we believe there is a gap in the life insurance industry's understanding of how climate change will impact their balance sheets in the medium to long-term. It is common industry practice today to run a number of 'climate change scenarios' through actuarial models as part of climate stress testing exercises. While this may be appropriate or sufficient for insurers to formulate their initial view on climate risk, we believe this approach is insufficient given the recent extreme weather events that have transpired globally including monsoon floods in Malaysia, bush fires in Australia and extreme heatwaves in the United States to name a few.

The purpose of this presentation is to introduce the actuarial community to our ongoing research on how extreme weather impacts human mortality. We start with a basic introduction to greenhouse gas emissions and how these emissions result in global warming. We then proceed to show 'evidence' of global warming in historical climate data and discuss whether this data can be used to establish a link between climate variables and insurance risks. Finally, we discuss which climate variables are relevant when studying the impact of heatwaves on human mortality and how climate data, such as individual weather station measurements, can be used to produce a 'heat index'.

Extreme Weather and Climate Effects on Infrastructure Maintenance - Rainhard Bengez

The extreme weather and climate effects we are facing continuously impact (public) infrastructure, such as embankments and pipes, subways or harbors, streets and buildings, bridges, networks, and so on, by reducing its functionality immediately or reducing its remaining technical (or economic) lifespan at higher rates directly or due to the effect of reduced maintenance because of the shifted budget from one infrastructure asset to the other, yet more affected infrastructure asset. Furthermore, their reliability is suffering, and because of the reallocation of budget, new infrastructure projects might be affected as well. The question of public or social benefit or use under constraints and with a limited budget arises. The decision problem is whether to invest in infrastructure or in technology to prevent damage to the existing infrastructure. By identifying the infrastructure (and its essential parts) in terms of asset allocation and portfolio management, we can apply concepts of optimal asset allocation in multi-period investment setups.

The first contribution of this paper is to express the infrastructure maintenance problem in terms of and concepts of asset allocation and portfolio management and to derive a control system theory-based model approach to identify the optimal asset allocation. Secondly, we apply frailty cluster models to gain insight into the hazard rates and changed reliability probabilities of the underlying infrastructure assets and portfolio.

Frailties were a concept introduced by Greenwood and Yule in 1920. The term "frailty" was introduced by Vaupel in 1979. Frailties are a convenient approach to considering unmeasured covariates such as random effects or unobserved heterogeneity in models of reliability (or survival) data.

In its basic form, "frailty" is a proportional factor modifying an object's hazard function. In general, they are extensions of the proportional hazard model (aka the Cox model in survival analysis, 1972). One can distinguish two broader classes of this model family. Firstly, models with univariate time endpoints and, secondly, multivariate endpoint models that consider dependence on clustered event times are suitable for recurrence of events, competing risks, etc.

The model does not assume any particular distribution of asset returns.

Modelling weather factors and applications to pricing and hedging weather-linked derivatives via Epstein-Zin utility

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We study the pricing of an agricultural insurance contract for an agent with recursive utility of Epstein-Zin type. We consider controlled systems that are modulated by an external factor. The climate factor considered is the temperature. First, we find the stochastic differential equation driving the dynamics of the temperature. Second, applying a stochastic maximum principle we compute the optimal strategies. Finally, we numerically compute the price of the contract.

TITLE Making Decisions under Model Misspecification

Fabio Maccheroni

ABSTRACT We use decision theory to confront uncertainty that is sufficiently broad to incorporate "models as approximations." We presume the existence of a featured collection of what we call" structured models" that have explicit substantive motivations. The decision maker confronts uncertainty through the lens of these models, but also views these models as simplifications, and hence, as misspecified. We extend the max-min analysis under model ambiguity to incorporate the uncertainty induced by acknowledging that the models used in decision-making are simplified approximations.

Formally, we provide an axiomatic rationale for a decision criterion that incorporates model misspecification concerns.

AUTHORS Simone Cerreia--Vioglio, Lars Peter Hansen, Fabio Maccheroni, and Massimo Marinacci

About pricing some complex life insurance products

Raghid Zeineddine.

Abstract: I would like to speak in my talk about pricing some complex life insurance products called variable annuities that represent three features: a minimum accumulation benefit to be paid at the maturity of the life insurance contract, a death benefit and a surrender benefit. Considering a model for this last benefit is very important for insurance companies especially after the covid-19 pandemic. The setup is based on a hybrid model for the financial market and uses time-inhomogeneous Levy processes as risk drivers. This topic has been studied in my following recently published paper: *Fourier based methods for the management of complex life insurance products. INSURANCE MATHEMATICS & ECONOMICS, 101, 320-341. doi:10.1016/j.insmatheco.2021.08.009.*

Thank you in advance,

Raghid Zeineddine.
Lecturer in Actuarial and Financial Mathematics at the University of Liverpool.

Title: Climate change impact upon mortality in actuarial models

Type: Oral presentation

Authors: Abdal Chaudhry (Barnett Waddingham), Michael Leitschkis (Milliman), Han Li (University of Melbourne), Qihe Tang (UNSW Sydney).

Intended speaker: Abdal Chaudhry, Michael Leitschkis, and Han Li

Abstract:

Based on the foundation discussed in the talk "Climate change science, extreme weather and mortality", we are going to examine how actuaries can account for climate impact upon mortality risk in actuarial projection models. Catastrophic weather events and mortality experience can simultaneously trigger large losses across different lines of insurance business. For life insurance business, "shocks" in climate change may trigger "shocks" in mortality experience. As a first step, we need a way of measuring dependencies between climate variables and mortality. Hence, we introduce a bivariate peak over threshold (POT) approach as the mathematical foundation for estimation of tail dependency coefficients, following up on prior research:

https://www.tandfonline.com/doi/abs/10.1080/10920277.2020.1823236?journalCode=uaaj20.

In this research, the extreme dependence between death counts and temperature indexes is estimated based on bivariate generalized Pareto distribution. The empirical study using monthly temperature and death data in the U.S. found that the joint extremes in cold weather and old-age death counts exhibit the strongest level of dependence.

We proceed by estimating the relevant dependency coefficients between UK weather station data series and UK population mortality data and interpreting them. Subsequently, we discuss how these results can be used in projection models without explicitly implementing climate variables in the latter. In particular, we outline how multivariate risk scenarios can be simulated consistently to our tail dependency estimates. Via this approach, it is then possible to apply proxy modelling techniques used in Solvency II Internal Model context – such as Least Squares Monte Carlo – in order to produce capital impact estimates. This is a possible way of bridging the gap between climate models and actuarial projection models, allowing for climate change impacts in the latter.

Subsidising Inclusive Insurance to Reduce Poverty

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In this talk, we consider a compound Poisson-type model for household capital and employ risk theory techniques to determine the probability that a household falls below the poverty line. Microinsurance is presented as a risk coping mechanism for the lower income classes. Specifically targeting low-income individuals living close to or below the poverty line, inclusive insurance, commonly referred to as microinsurance, aims to close the protection gap that exists between uninsured and insured losses to life, property and health by providing protection to the poor. However, barriers to microinsurance penetration exist due to constraints on product affordability resulting from fundamental features of the microinsurance environment. Premium payments can in fact heighten the risk of falling into poverty for the proportion of the population living just above the poverty line. Our preliminary results validate those previously obtained with this type of model, showing that microinsurance alone is not sufficient to reduce the probability of falling into the area of poverty for specific household groups, since premium payments constrain household capital growth. This indicates the need for additional aid, particularly from the government.

As such, we analyse several premium subsidy strategies and discuss the role of government in subsidising microinsurance to help reduce poverty. We assume households are subject to capital shocks of random size, due to, for example, severe illness, the death of a household member or breadwinner and catastrophic climate-related events such as floods and earthquakes. Obtaining explicit solutions for the trapping probability, we compare the influence of microinsurance frameworks on the ability of households to stay above the poverty line. Aligning with the need for an alternative solution for insurance driven social protection, we assess for the first time in the non-ruin context, to the best of our knowledge, the impact of a (government) subsidised microinsurance scheme with a barrier strategy, where households are required to pay premiums only when their capital is above a given level.

Keywords: microinsurance; poverty traps; trapping probability; social protection; government subsidies.

3rd IMA Conference on Mathematics of Finance (and Climate Change Risk)
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Balancing Adaptation and Mitigation in long term Climate Policy

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Environmental sustainability is one of the major challenges in the modern world. Environmental adaptation and pollution mitigation are two main long-term policy instruments to combat negative impacts of climate change. Both of them are extremely important and interconnected, though require enormous investments. However, all countries possess different means that depend on country's economic efficiency and stage of development. Mitigation and adaptation are thoroughly considered while designing long-term policies and regulations on both international and national levels.

A simple analytic modelling framework for rational investment to adaptation and mitigation in symmetric and asymmetric multi-country world is presented. A suggested economic-environmental model aims to analyze strategic behavior of several countries engaged in capital accumulation, pollution mitigation, and environmental adaptation in the context of an environmental common good. Focusing on economic inequalities, environmental vulnerability and possible international transfer among countries, a range of alternatives that ensure positive payoffs for a multi-region world is discussed. The influence of country asymmetry on modelling outcomes and policy implications is revealed. Comparison of competitive and cooperative strategies is analyzed.

The interpretation of obtained modelling outcomes leads to relevant policy recommendations and financial implications for individual countries. It contributes to integration between international and domestic climate policies and improvement of resilience and sustainability of the global world.

The Carbon Equivalence Principle

Andrea Macrina

Sustainability, focusing on climate change, is a key issue for financial market participants. WWF International, among many other global organisations, emphasise the crucial role financial organisations and systems play in decarbonising human life and in enabling drastic reduction of adverse impact to our planet. Alignment of financial market incentives and carbon emissions disincentives is key to limiting global warming. Regulators and standards bodies have made a start by requiring some carbon-related disclosures and proposing others. In our paper we go further and propose a Carbon Equivalence Principle: all financial products shall contain a description of the equivalent carbon flows from greenhouse gases that the products enable, as well as their existing description in terms of cash flows. This description of the carbon flows enabled by the project shall be compatible with existing bank systems that track cashflows so that carbon flows have equal standing to cash flows. We demonstrate that this transparency alone can align incentives by applying it to project finance examples for power generation and by following through the financial analysis. The financial requirements to offset costs of carbon flows enabled in the future radically change project costs, and risk that assets become stranded, thus further increasing costs. This observation holds whichever partner in the project bears the enabled-carbon costs. Mitigating these risks requires project re-structuring to include negative emissions technologies. We also consider that sequestered carbon needs to remain sequestered permanently, e.g., for at least one hundred years. The adoption of the Carbon Equivalence Principle for financial products aligns incentives, requires product redesign, and is simply good financial management driving sustainability.

Title: Systemic Risk in Markets with Multiple Central Counterparties
Luitgard Veraart
Abstract:
We provide a framework for modelling risk and quantifying payment shortfalls in cleared markets with multiple central counterparties (CCPs). Building on the stylised fact that clearing membership is shared among CCPs, we show that stress in this shared membership can transmit across markets through multiple CCPs. We provide stylised examples to lay out how such stress transmission can take place, as well as empirical evidence to illustrate that the mechanisms we study could be relevant in practice. Furthermore, we show how stress mitigation mechanisms such as variation margin gains haircutting by one CCP can have spillover effects to other CCPs. The framework can be used to enhance CCP stress-testing, which currently relies on the ``Cover 2'' standard requiring CCPs to be able to withstand the default of their two largest clearing members. We show that who these two clearing members are can be significantly affected by higher-order effects arising from interconnectedness through shared clearing membership.
This is joint work with Iñaki Aldasoro (Bank for International Settlements).

Stochastic optimal control for SDEs with rough coefficients.

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Abstract

In this talk, we consider stochastic optimal control of systems driven by stochastic differential equations with irregular drift coefficient. Such phenomenon can be observed in a market with regime switching. We establish a necessary and sufficient stochastic maximum principle. To achieve this, we first derive an explicit representation of the first variation (in the Sobolev sense) process of the controlled diffusion. Since the drift coefficient is not smooth, the representation is given in terms of the local time of the diffusion process. Then by an approximation argument, we construct a sequence of optimal control problems with smooth coefficients. Finally, we use Ekeland's variational principle to obtain an approximating adjoint process from which we derive the maximum principle by passing to the limit. The

This talk is based on a joint work with L. Tangpi.

Keywords: Ekeland's variational principle; first variation process; maximum principle; measurable drift coefficients.

Stochastic volatility models and data-science-driven improvements

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February 9, 2022

Abstract

In this presentation, at first, we discuss the Barndorff-Nielsen and Shephard (BN-S) model, a stochastic volatility model that is useful for both derivative and commodity markets. Though this model is very efficient and analytically tractable, it is well known that it suffers from the absence of long-range dependence and many other issues. We discuss some generalizations of this model. It will be shown that a further generalization is possible with the implementation of various machine/deep learning algorithms. This resulting refined BN-S model is more efficient and has fewer parameters than the superposition models that are used in practice to improve the BN-S model. An application of this refined BN-S model will be presented to find an optimal hedging strategy for the oil commodity from the Bakken, a new region in the United States of oil extraction that is benefiting from fracking technology.

Fokker-Planck equations representing generalised Ornstein-Uhlenbeck processes (including multi dimensions)

Abigail Mellor

With an array of applications in financial mathematics, including some linked to energy pricing issues and option pricing, we present a dissection of factors which attribute to an efficient finite differencing scheme for the Fokker-Planck equations for Ornstein-Uhlenbeck stochastic processes. An Ornstein-Uhlenbeck (OU) process is a diffusion process originally introduced in the theory of particle motion and in current times have been used by practitioners in the modelling of interest rates and commodity prices (examples such as the Vasicek, CIR and Chen[1] model). Another use is found in the realm of stochastic volatility; Heston [2] posed the volatility process as an OU process where the balance between the drift and non-deterministic term create a dampening effect on the volatility rate. The Fokker-Planck equation describes the forward evolution of the probability density of the OU process through time and can be used, to give an example in a financial mathematics scenario, to trace the probabilistic evolution of an asset's price in order to correctly value dependent financial contracts. Consider a generalised OU process given by

$$dx_n = \kappa_n(\theta_n - x_n)dt + \sum_{m=1}^{N} \sigma_n x_{nm}^{\gamma_n} dW_m, \qquad n = 1, 2, \ldots, N \tag{1}$$

where dW_n are Brownian motions. The N-dimensional Fokker-Planck Equation for the given generalised OU processes is

$$\frac{\partial u}{\partial t} + \sum_{n=1}^{N} \left(\frac{\partial}{\partial x_n}(\kappa_n(\theta_n - x_n)u) \right) - \tfrac{1}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \left(\frac{\partial^2}{\partial x_n \partial x_m}(\sigma_n \sigma_m \rho_{nm} x_n^{\gamma_n} x_m^{\gamma_m} u) \right) = 0 \tag{2}$$

where $\rho_{mm} = 1$, $\rho_{nm} = \rho_{mn}$ and $b_{nm} = \sigma_n x^{\gamma_n}$.

In certain parameter regimes (notably, but not exclusively those involving fractional values of the γ_n), the solution structure can be quite intricate, and standard computational methods for such OU problems fail. Indeed, it is worth noting that the aforementioned fractional values are often observed when calibrating real market prices. The nature of this structure is first described, using rigorous, mathematical (asymptotic) ideas, and these are then built into the numerical schemes we use to tackle these OU processes. The net result is robust numerical solutions (as confirmed by extensive numerical scheme experimentation), obtained efficiently with modest computational facilities.

References

[1] Lin Chen. *Stochastic mean and stochastic volatility: a three-factor model of the term structure of interest rates and its applications in derivatives pricing and risk management.* Blackwell publishers, 1996.

[2] Steven L Heston. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *The review of financial studies*, 6(2):327–343, 1993.

Poster presentation

THE SECURITISATION OF THE EUROZONE SOVEREIGN DEBT

Sylvia Gottschalk
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This paper investigates the risk implications of securitising the Eurozone sovereign debt as collateralised debt obligations (CDO). The proposal of creating asset-backed securities underpinned by Eurozone sovereign bonds has gained traction since the Eurozone Sovereign Debt Crisis of 2011-2014, and particularly in the aftermath of the Covid-19 pandemic. It is being seriously considered by the European Union as an alternative to Eurobonds, which are strongly opposed by Eurozone members that reject the mutualisation of sovereign debt[1]. Under these proposals, the senior tranche of the CDO would have the risk profile of a sovereign bond.

This paper constructs a structural model of a synthetic CDO where the underlying comprises the government bonds of all the 19 Eurozone countries, weighted by their respective countries' GDP. The default correlation structure of the portfolio loss is modelled as a Bernoulli mixture where default risk depends on several stochastic common factors. Modelling the portfolio loss distribution of a Eurozone CDO presents two specific challenges. Firstly, the default probabilities of individual sovereigns cannot be assumed to be equal and independent, as is common in credit risk modelling. Consequently, the portfolio loss distribution must be that of the sum of non-iid Bernoulli random variables. Secondly, owing to the fact that the portfolio only includes the bonds of 19 Eurozone sovereigns, the probability distribution of its loss cannot be approximated by an asymptotic distribution that is often used in large homogeneous credit portfolios. Obtaining a tail probability from the sum of non-iid Bernoulli distributions is computationally intractable even for a relatively small number of assets, but its cumulant generating function is relatively easy to compute. Based on these cumulants, the expected portfolio loss, and thus, the expected tranche loss of the CDO can be accurately calculated by saddlepoint approximations. Numerical experiments show that only a very high subordination level, where the junior tranche absorbs 30% of portfolio losses ensures that the senior tranche does not default. In comparison, the highest subordination level of an iTraxx CDO is 22%.

[1] https://www.esrb.europa.eu/pub/task_force_safe_assets/shared/pdf/esrb.report290118_sbbs_volume_II_technicalanalysis.en.pdf