Mixed-Integer Convex Optimization for Statistical Learning

Many statistical learning problems involving sparsity and other structural constraints can be formulated as mixed-integer convex optimization problems involving indicator variables and constraints on these indicators. As motivation, the first part of this talk will focus on the problem of learning directed acyclic graphs (DAGs) from continuous observational data, also known as the causal discovery problem. Current state-of-the-art structure learning methods for this problem face significant limitations: (i) they lack optimality guarantees and often yield suboptimal solutions; (ii) they rely on the restrictive assumption of homoscedastic noise. To address these shortcomings, we propose a computationally efficient mixed-integer programming framework. Numerical experiments demonstrate that our method outperforms existing algorithms and is robust to noise heteroscedasticity.

In the second part of the talk, we address the challenge of weak continuous relaxations inherent in natural mixed-integer convex formulations for such structured learning problems, which hinder the performance of branch-and-bound-based solvers. We develop novel methods to strengthen these formulations by deriving a convex hull description of the associated mixed-integer set in an extended space. Notably, this approach reduces the convexification of these problems to the characterization of a polyhedral set in the extended formulation. Our new theoretical framework unifies several previously established results and provides a foundation for applying polyhedral methods to strengthen the formulations of such mixed-integer nonlinear sets.